2 SEM TDC MTH M 1

2020

MATHEMATICS

(Major)

Course : 201

(Matrices, Ordinary Differential Equations, Numerical Analysis)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Matrices)

(*Marks* : 20)

1. (a) Write the rank of a matrix whose every element is unity. 1

   (b) Choose the correct answer for the following : 1

   Let r be the rank of a matrix A. Then every square submatrix of order $r+1$ is a

   (i) null matrix

   (ii) singular matrix

(iii) non-singular matrix

(iv) row matrix

(c) Find the rank of

$$\begin{bmatrix} 1 & 1 & 1 \\ k & k & k \\ k^2 & k^2 & k^2 \end{bmatrix}, k \in \mathbb{R}^+$$ 2

(d) Find the rank of the matrix A by reducing it to canonical form, where

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$$ 4

Or

Find the rank of the matrix

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 3 & 2 & -4 & 1 \\ 5 & -3 & 2 & 6 \end{bmatrix}$$

by reducing it to echelon form.

2. (a) Write the condition when a system $Ax = b$ of m linear equations in n unknowns is consistent. 1

(b) If k is an eigenvalue of a non-singular matrix A, then write an eigenvalue of A^{-1}. 1

(c) Show that the following system of equations

$$\begin{aligned} x+y+z &= 9 \\ 2x+5y+7z &= 52 \\ 2x+y-z &= 0 \end{aligned}$$

is consistent and hence solve it. 5

Or

Show that the following equations

$$\begin{aligned} 2x-y+z &= 4 \\ 3x-y+z &= 6 \\ 4x-y+2z &= 7 \\ -x+y-z &= 9 \end{aligned}$$

are inconsistent.

(d) Show that the matrix

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

satisfies its own characteristic equation. 5

Or

Determine the eigenvalues of the matrix

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -2 & -2 & 0 \end{bmatrix}$$

Group—B

(Ordinary Differential Equations)

(*Marks* : 30)

3. *(a)* If

$$\frac{dy}{dx} + py = 0$$

then write the solution of the equation. 1

(b) If M and N are both homogeneous functions of x and y of degree n of the differential equation $Mdx + Ndy = 0$, then write an integrating factor of the equation. 1

(c) Solve any *one* of the following : 4

(i) $\frac{dy}{dx} + x\sin 2y = x^3 \cos^2 y$

(ii) $x\frac{dy}{dx} - 2y = xy^4$

(d) Solve $(x^2 + y^2)dx - 2xydy = 0$ 4

Or

Show that e^x, e^{-2x}, e^{2x} are linearly independent solutions of a differential equation. Also, find the differential equation.

4. (a) Let the transformation $x = e^z$ be applied to the equation

$$a_0 x^2 \frac{d^2y}{dx^2} + a_1 x \frac{dy}{dx} + a_2 y = F(x),\ a_0, a_1, a_2 \in R$$

Write the general form of the transformed differential equation. 2

(b) Solve any *two* from the following : 4×2=8

(i) $\frac{d^3y}{dx^3} + \frac{dy}{dx} = 2x^2 + 4\sin x$

(ii) $\frac{d^2y}{dx^2} - y = 3x^2 e^x$

(iii) $x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 4\log x$

5. Solve any *two* of the following : 5×2=10

(i) $\frac{d^2y}{dx^2} + y = \tan x$, by using variation of parameters

(ii) $(1+x^2)^2 \frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} + 4y = 0$, by changing independent variable

(iii) $\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} - n^2 y = 0$, by removing the first order derivative

GROUP—C

(Numerical Analysis)

(*Marks* : 30)

6. *(a)* State True or False : 1

A transcendental equation may have no roots.

(b) Write how many function evaluations require per iteration in secant method. 1

(c) Explain the geometrical interpretation of Newton-Raphson method. 4

(d) Find a real root of the equation $x^3 - x - 4 = 0$ lying between 1 and 2 by using bisection method (perform 3 iterations). 5

Or

Describe Gauss-Seidel method.

(e) Find the reciprocal of $\frac{1}{7}$ by using Newton-Raphson method. 4

Or

Solve

$$\begin{aligned} x_1 + x_2 - x_3 &= 2 \\ 2x_1 + 3x_2 + 5x_3 &= -3 \\ 3x_1 + 2x_2 - 3x_3 &= 6 \end{aligned}$$

by using Gauss elimination method.

7. *(a)* Show that $\delta(x_0, x_1) = \delta(x_1, x_0)$. 1

(b) Show that $\Delta - \nabla = \Delta \nabla$. 2

(c) If $f(x) = e^{ax}$, find $\Delta^2 f(x)$. 2

(d) Deduce Newton's general interpolation formula. 6

Or

Evaluate $\int_0^{10} x^2 dx$ by using Simpson's $\frac{1}{3}$ rule.

(e) Deduce trapezoidal rule for numerical integration. 4

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