2 SEM TDC MTH M 1

2019

(May)

MATHEMATICS

(Major)

Course : 201

(Matrices, Ordinary Differential Equations, Numerical Analysis)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Matrices)

(*Marks* : 20)

1. *(a)* Choose the correct option : 1

If a matrix A has a non-zero minor of order r, then

(i) $\text{rank}(A) = r$

(ii) $\text{rank}(A) \geq r$

(iii) $\text{rank}(A) < r$

(iv) $\text{rank}(A) \leq r$

(*b*) For what value of x the rank of the matrix

$$A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$$

will be less than 3? 2

(*c*) Reduce the matrix A to its normal form where

$$A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

Hence, find the rank of A. 5

Or

Reduce the following matrix into echelon form and find its rank :

$$\begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}$$

2. (*a*) Under what condition a system of m homogenous linear equations $AX = 0$ in n unknowns will possess infinite number of solutions? 1

(b) For what value of k the system of equations

$$\begin{aligned} x+5y-3z &= -4 \\ -x-4y+z &= 3 \\ -2x-7y &= k \end{aligned}$$

is consistent? Solve it. 5

(c) Show that the matrix

$$A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$$

satisfies Cayley-Hamilton theorem. Hence, compute A^{-1}. 4+2=6

Or

What is the degree of characteristic polynomial of an $n \times n$ square matrix? Determine the characteristic roots and characteristic vectors of the matrix

$$A=\begin{bmatrix}3 & 5\\ -2 & -4\end{bmatrix}$$

1+5=6

GROUP—B

(Ordinary Differential Equations)

(*Marks* : 30)

3. *(a)* Write the general solution of the differential equation

$$\frac{d^3y}{dx^3} = 0$$

if 1, x, x^2 are its linearly independent solutions. 1

(b) Solve : 2

$$(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$$

(c) Find the general solution of the differential equation $p = \tan(px - y)$, where

$$p = \frac{dy}{dx}$$ 3

(d) Answer any *one* of the following : 4

(i) Evaluate Wronskian of the functions e^x and xe^x. Hence, conclude whether or not they are linearly independent. If they are independent, set up the differential equation having them as its independent solutions.

(ii) Solve :

$$(x^2 + y^2 + x)\,dx - (2x^2 + 2y^2 - y)\,dy = 0$$

4. (a) Under what condition $y = x$ is a part of the complementary function of the differential equation

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R \text{ ?}$$

1

(b) Find the particular integral of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = x$$

2

(c) Answer any *one* of the following : 3

(i) Solve :

$$\frac{d^2y}{dx^2} + 4y = x\cos x$$

(ii) Solve :

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2e^{3x}$$

(*d*) Answer any *one* of the following : 4

(*i*) Solve :

$$(x^2D^2 + xD + 1)y = \sin\log x^2,$$

where $D \equiv \frac{d}{dx}$

(*ii*) Solve

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 9y = 0$$

given that $y = x^3$ is a solution.

5. Answer any *two* of the following : 5×2=10

(*a*) Solve by removal of the first-order derivative :

$$\frac{d^2y}{dx^2} - 2\tan x\frac{dy}{dx} + 5y = 0$$

(*b*) Solve by changing the independent variable :

$$x\frac{d^2y}{dx^2} + (4x^2 - 1)\frac{dy}{dx} + 4x^3y = 2x^3$$

(*c*) Solve by the method of variation of parameters :

$$\frac{d^2y}{dx^2} + a^2y = \operatorname{cosec} ax$$

where a is a constant.

GROUP—C

(Numerical Analysis)

(*Marks* : 30)

6. (*a*) State True or False : 1

Iteration method is always convergent.

(*b*) Evaluate $\sqrt{12}$ using Newton-Raphson method by performing two iterations. 4

Or

Describe Newton-Raphson method for finding real roots of an algebraic equation.

(*c*) Find the real root of the equation $x^3 - x - 1 = 0$ lying between 1 and 2 using bisection method by performing three iterations. 5

Or

Find a real root of the equation $x^3 - 2x - 5 = 0$ using regula-falsi method by performing three iterations.

(*d*) Solve by Gauss elimination method : 5

$$2x + 2y + 4z = 14$$
$$3x - y + 2z = 13$$
$$5x + 2y - 2z = 2$$

Or

Solve by Gauss-Seidel method by performing two iterations :

$$5x+2y+z=12$$
$$x+4y+2z=15$$
$$x+2y+5z=20$$

7. (a) What is the degree of the interpolating polynomial in Simpson's $\frac{3}{8}$ rule? 1

(b) Show that $(1+\Delta)(1-\nabla)=1$, where the symbols have their usual meanings. 2

(c) If $f(x)=\dfrac{1}{x^2}$, find the divided difference $f(a, b)$. 2

(d) Answer any *two* of the following questions : 5×2=10

(i) Derive Newton's forward interpolation formula.

(ii) The population of a town is as follows :

Year	x	:	1891	1901	1911	1921	1931
Population in lakh	y	:	46	66	81	93	101

Estimate the population for the year 1925.

(iii) Deduce Simpson's $\frac{1}{3}$ rule for numerical integration.

(iv) Find the form of the function given by

x	:	1	2	5
$f(x)$	:	1	4	10

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