2 SEM TDC MTH M 1

2018

(May)

MATHEMATICS

(Major)

Course : 201

(Matrices, Ordinary Differential Equations, Numerical Analysis)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Matrices)

(*Marks* : 20)

1. *(a)* If A is an n-rowed non-singular matrix, then what is the rank of A^T? 1

   (b) Find the rank of the matrix 2

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 0 & 1 & 1 \end{pmatrix}$$

(c) Reduce the matrix A to its normal form where

$$A=\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}.$$

Hence find the rank of A. 5

Or

Find the rank of the matrix

$$\begin{pmatrix} 1 & 1 & -3 & 2 \\ 2 & -1 & 2 & -3 \\ 3 & -2 & 1 & -4 \\ -4 & 1 & -3 & 1 \end{pmatrix}$$

by reducing it to echelon form.

2. Answer any *two* of the following : 6×2=12

(a) Define characteristic roots of a matrix. Find the characteristic equation of the matrix

$$A=\begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{pmatrix}$$

and verify Cayley-Hamilton theorem. Hence compute A^{-1}.

(b) Find for what values of λ, the equations

$$x+y+z=1$$
$$x+2y+4z=\lambda$$
$$x+4y+10z=\lambda^2$$

have a solution and also solve them completely in each case.

(c) What do you mean by homogeneous and non-homogeneous linear equations? Show that the system of equations

$$5x+3y+7z=4$$
$$3x+26y+2z=9$$
$$7x+2y+10z=9$$

is consistent and solve it.

GROUP—B

(Ordinary Differential Equations)

(*Marks* : 30)

3. (a) Find Wronskian of $\cos bx$ and $\sin bx$ $(b \neq 0)$. 1

(b) Solve : 2

$$(x+y+1)\frac{dy}{dx}=1$$

(c) Find the complete solution and singular solution of the differential equation

$$y = px + f(p), \text{ where } p = \frac{dy}{dx}$$ 3

(d) Answer any *one* of the following : 4

(i) Solve :

$$xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$$

(ii) Prove that Wronskian of the functions $e^{m_1 x}$, $e^{m_2 x}$, $e^{m_3 x}$ is equal to

$$(m_1 - m_2)(m_2 - m_3)(m_3 - m_1)e^{(m_1 + m_2 + m_3)x}$$

4. *(a)* Under what condition $y = e^{ax}$ will be a solution of the equation

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0 ?$$ 1

(b) Show that the roots of the auxiliary equation are 1, 1, –2 of the differential equation

$$x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^2$$ 2

(c) Solve any *one* of the following : 3

(i) $\frac{d^2y}{dx^2}+y=\cos 2x$

(ii) $(D^2-4D+4)y=x^3e^{2x}$

(d) Solve any *one* of the following : 4

(i) $(x^2D^2-3xD+5)y=\sin(\log x)$ where $\frac{d}{dx}\equiv D$

(ii) $\sin^2 x\cdot\frac{d^2y}{dx^2}=2y$, given $y=\cot x$ is a solution

5. Answer any *two* of the following : 5×2=10

(a) Solve by removal of the first-order derivative :

$$\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+(x^2+1)y=x^3+3x$$

(b) Solve by changing the independent variable :

$$\frac{d^2y}{dx^2}+\cot x\frac{dy}{dx}+4y\,\mathrm{cosec}^2 x=0$$

(c) Solve by the method of variation of parameters :

$$\frac{d^2y}{dx^2}+9y=\sec 3x$$

GROUP—C

(Numerical Analysis)

(*Marks* : 30)

6. (a) What is the degree of convergence of the Newton-Raphson method? 1

(b) Give the geometrical interpretation of Newton-Raphson method. 4

(c) Solve $x^3-2x-5=0$ for the positive root by iteration method. 5

Or

Solve the equation $x\tan x+1=0$ by regula falsi method starting with $a=2\cdot5$ and $b=3$ correct to 3 decimal places.

(d) Solve by Gauss elimination method : 5

$$2x+3y-z=5;\ 4x+4y-3z=3;\ 2x-3y+2z=2$$

Or

Apply Gauss-Jordan method to find the solution of the following system :

$$10x+y+z=12;\ 2x+10y+z=13;\ x+y+5z=7$$

7. (a) State 'true' or 'false' : 1

Simpson's one-third rule is better than the trapezoidal rule.

(b) Show that $\delta \equiv E^{1/2} - E^{-1/2}$, where the symbols have their usual meanings. 2

(c) Evaluate : 2

$$\Delta^3 (1-x)(1-2x)(1-3x) \text{ if } h = 1$$

(d) Answer any *two* of the following : 5×2=10

(i) The population of a town is as follows :

Year	x :	1941	1951	1961	1971	1981	1991
Population in Lakhs	y :	20	24	29	36	46	51

Estimate the population increase during the period 1946 to 1976.

(ii) Evaluate

$$\int_0^1 \frac{dx}{1+x}$$

by dividing the range into 10 equal parts correct to four decimal places.

(iii) Derive the Newton's forward interpolation formula.

(iv) Deduce the general quadrature formula for equidistant ordinates.

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