

2016

(May)

MATHEMATICS

(Major)

Course : 201

(**Matrices, Ordinary Differential Equations,
Numerical Analysis**)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

A : Matrices

(Marks : 20)

1. (a) State whether True or False : 1
Rank of a matrix is a positive integer.
- (b) Define elementary transformations of matrices. 2
- (c) Show that rank of the product of two matrices cannot exceed that of either matrix. 5

(2)

2. (a) Show that the following equations are consistent and solve them by matrix method : 6

$$x+2y+3z=14$$

$$3x+y+2z=11$$

$$2x+3y+z=11$$

Or

State and prove Cayley-Hamilton theorem. 6

- (b) Find the characteristic values and characteristic vectors of the following matrix : 6

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

B : Ordinary Differential Equations

(Marks : 30)

3. (a) Find the integrating factor of the differential equation

$$\frac{dy}{dx} + Py = Q$$

P and Q are functions of x. 1

(b) Solve (any two) : 3×2=6

(i) $x \frac{dy}{dx} + 2y = x^2 \log x$

(ii) $\frac{dy}{dx} = x^3 y^3 - xy$

(iii) $y = px + \frac{a}{p}; p = \frac{dy}{dx}$

(c) Show that the solutions $\sin x$ and $\cos x$ of

$$\frac{d^2 y}{dx^2} + y = 0$$

are linearly independent. 3

4. (a) Solve : 2

$$(D^4 + 2D^3 + D^2)y = 0; \text{ where } D = \frac{d}{dx}$$

(b) Solve (any two) : 4×2=8

(i) $(D^3 - 2D + 4)y = e^x \cos x; D = \frac{d}{dx}$

(ii) $(D^4 + 2D^2 + 1)y = x \cos x$

(iii) $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4$

5. (a) Describe the method of removal of the first derivative of the differential equation

$$\frac{d^2 y}{dx^2} + P \frac{dy}{dx} + Qy = R \quad 5$$

(b) Solve (any one) : 5

(i) $x^2 \frac{d^2y}{dx^2} - 2x(1+x) \frac{dy}{dx} + 2(1+x)y = x^3$

(ii) $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$

by putting $z = \sin x$.

C : Numerical Analysis

(Marks : 30)

6. (a) Write the condition of convergence of iteration method. 1

(b) In solving system of linear algebraic equation, what are the differences between 'Gauss elimination method' and 'Jordan method'? 2

(c) Find a real root of the equation $x^3 - 4x - 9 = 0$ by using bisection method correct to three decimal places. 6

Or

Find a root of the equation $x^3 - 2x^2 - 5 = 0$ by using Newton-Raphson method correct to three decimal places. 6

- (d) Solve the following equations by Gauss-Jordan method : 6

$$x + 2y + z = 8$$

$$2x + 3y + 4z = 20$$

$$4x + 3y + 2z = 16$$

7. (a) Define interpolation. 1
(b) Evaluate $\Delta^2 x^3$. 2
(c) Answer (any two) : $6 \times 2 = 12$

(i) Deduce 'Newton's forward interpolation formula'.

(ii) Derive Simpson's one-third rule for numerical integration.

(iii) Evaluate :

$$\int_1^2 \frac{dx}{x}$$

by Simpson's $\frac{3}{8}$ th rule.
