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2 SEM TDC MTH M 1

2015

(May)

MATHEMATICS

(Major)

Course : 201

(Matrices, Ordinary Differential Equations, Numerical Analysis)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

A : Matrices

(*Marks* : 20)

1. *(a)* Under what condition, the rank of the following matrix A is 3? 1

$$A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$$

(b) Show that the rank of a skew-symmetric matrix cannot be one. 2

(c) Reduce the matrix A to its normal form and hence find its rank : 5

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

Or

Find non-singular matrices P and Q such that PAQ is in the normal form, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

2. *(a)* Under what condition, a system of m homogeneous linear equations $AX = 0$ in n unknowns has only trivial solution? 1

(b) What is the eigenvalue of $P^{-1}AP$, if eigenvalue of matrix A is λ? 1

(c) Investigate for what values of λ and μ, the simultaneous equations

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

have *(i)* no solution, *(ii)* a unique solution and *(iii)* an infinite number of solutions. 4

(d) State and prove Cayley-Hamilton theorem. 6

Or

Find the characteristic roots and associated characteristic vectors for the matrix

$$A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

B : Ordinary Differential Equation

(*Marks* : 30)

3\. *(a)* Is the differential equation $(2x-y+1)\,dx+(2y-x+1)\,dy=0$ exact? 1

(b) Find the integrating factor of the differential equation

$$x\cos x\frac{dy}{dx}+y(x\sin x+\cos x)=1$$ 2

(c) Solve (any *one*) : 3

(i) $(x+2y^3)\frac{dy}{dx}=y$

(ii) $x=y-p^2$

(d) If $y_1(n)$ and $y_2(n)$ are any two solutions of $a_0(n)y''(n)+a_1(n)y'(n)+a_2(n)y(n)=0$, then prove that the linear combination $c_1y_1(n)+c_2y_2(n)$, where c_1 and c_2 are constants, is also a solution of the given equation. 4

Or

Show that linearly independent solutions of $y''-2y'+2y=0$ are $e^x \sin x$ and $e^x \cos x$. What is the general solution? Find the solution $y(n)$ with the property $y(0)=2$, $y'(0)=-3$.

4. *(a)* If auxiliary equation has two equal pairs of imaginary roots, then what is the general solution of the second-order linear differential equation? 1

(b) What is the value of

$$\frac{1}{f(D^2)}\sin ax$$

if $f(-a^2)\neq 0$? 1

(c) Solve (any *two*) : 4×2=8

(i) $(D^2 - 4D + 13)y = 0, \; D \equiv \frac{dy}{dx}$

(ii) $(D^4 + 2D^2 + 1)y = x^2 \cos x$

(iii) $(D-2)^2 y = x^2 e^{3x}$

(d) Solve (any *two*) : 5×2=10

(i) $\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2} \sin 2x$

(by removing first-order derivative)

(ii) $\frac{d^2y}{dx^2} + \frac{3}{x}\frac{dy}{dx} + \frac{a^2}{x^6}y = \frac{1}{x^8}$

(by changing the independent variable)

(iii) $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$

(by applying the method of variation of parameters)

C : Numerical Analysis

(*Marks* : 30)

5. (a) Fill in the blank : 1

If $f(x)$ is continuous in the interval $[a, b]$ and if $f(a)$ and $f(b)$ are of opposite signs, then the equation $f(x) = 0$ will have —— real root between a and b.

(b) What is the length of the subinterval which contains x_n after n bisections? 1

(c) Using regula falsi method, find the first approximate value of the root of the equation $f(x) = x \tan x + 1$ that lies between 2·5 and 3. 3

(d) Answer (any *two*) : 5×2=10

(i) Describe Newton-Raphson method for obtaining the real roots of the equation $f(x) = 0$.

(ii) Apply Gauss-Jordan method, to find the solution of the following system :

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$x + y + 5z = 7$$

(iii) Solve by Gauss-Seidel method

$$28x + 4y - z = 32$$
$$2x + 17y + 4z = 35$$
$$x + 3y + 10z = 24$$

6. *(a)* State trapezoidal rule. 1

(b) Show that $E = e^{hD}$, where the symbols have their usual meanings. 2

(c) Evaluate $\Delta\left(\frac{2^x}{x!}\right)$. 2

(d) Answer (any *two*) : 5×2=10

(i) Deduce the Simpson's one-third rule.

(ii) The population of a town is as follows :

Year (x)	1941	1951	1961	1971	1981	1991
Population (in lakhs) (y)	20	24	29	36	46	51

Estimate the population increase during the period 1946 to 1976.

(iii) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by trapezoidal rule.

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