**2 SEM TDC STS M 1 (N/O)**

**2016**

( May )

STATISTICS

( Major )

Course : 201

**( Mathematics for Statistics—I )**

*The figures in the margin indicate full marks for the questions*

( New Course )

*Full Marks* : 48

*Pass Marks* : 14

*Time* : 2 hours

**1.** Choose the correct answer : 1×6=6

(a) If $A = \{1, 2, \{3, 4\}, 5\}$, then which of the following statements is incorrect?

(i) $\{3, 4\} \in A$

(ii) $\{\{3, 4\}\} \subset A$

(iii) $\{3, 4\} \subset A$

(iv) None of the above

(b) Which of the following is not equivalent to $A \subset B$?

(i) $A - B = \phi$

(ii) $A \cap B = A$

(iii) $A \cup B = B$

(iv) None of these

(c) If $S_{n+1} \geq S_n$, then the sequence $\{S_n\}$ is

(i) monotonic increasing

(ii) strictly increasing

(iii) monotonic decreasing

(iv) oscillatory

(d) According to Cauchy's root test, $\lim_{n \to \infty} (u_n)^{\frac{1}{n}} = l > 1$ means that the series $\Sigma u_n$ is

(i) convergent

(ii) divergent

(iii) oscillatory

(iv) convergent to 1 only

*(e)* The first derivative of the function $x^8$ w.r.t. another function $x^3$ is

*(i)* $\frac{3}{8}x^5$

*(ii)* $\frac{8}{3}x^5$

*(iii)* $24x^5$

*(iv)* None of the above

*(f)* The value of $\int_0^{\pi/2} \sin^6 x \, dx$ is

*(i)* $5\pi/64$

*(ii)* $5\pi/32$

*(iii)* $5/32$

*(iv)* None of the above

**2.** *(a)* If $S$ and $T$ are subsets of real numbers, then show that $(S \cup T)' = S' \cup T'$. 3

*(b)* Show that a set is closed iff its complement is open. 3

**3.** Answer any *two* of the following : 6×2=12

*(a)* Define a bounded sequence. If $\{a_n\}$ is a bounded sequence such that $a_n > 0$ for all $n \in N$, then show that

$$\underline{\lim}\left(\frac{1}{a_n}\right) = \frac{1}{\overline{\lim}\, a_n}, \text{ if } \overline{\lim}\, a_n > 0$$

1+5=6

*(b)* State Cauchy's first theorem on limits. Using the theorem, show that

$$\lim_{n \to \infty} \left[ \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}} \right] = 1$$

1+5=6

*(c)* What is monotonic sequence? Show that the sequence $\{a_n\}$ defined by

$$a_{n+1} = \frac{1}{2}\left(a_n + \frac{9}{a_n}\right), \; n \geq 1 \text{ and } a_1 > 0$$

converges to 3. 1+5=6

**4.** *(a)* Show that the function $f(x) = x^2 - 6x$ is increasing for $x > 3$. 2

*(b)* Show that $D^n(x^n) = n!$ 2

**5.** Answer any *two* of the following :

*(a)* If $\sin y = x \sin(a+y)$, then prove that

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

5

(b) If $z = \dfrac{x^2 y^2}{x+y}$, then prove that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3z$$ 5

(c) State and prove Leibnitz theorem. 5

6. Answer any *two* of the following :

(a) If $f(x) = f(a+x)$, then prove that

$$\int_0^{na} f(x)\,dx = n\int_0^{a} f(x)\,dx$$ 5

(b) Evaluate : 5

$$\int_0^2 \int_0^{\sqrt{4+x^2}} \frac{dx\,dy}{4+x^2+y^2}$$

(c) If $I_n = \int_0^{\pi/4} \tan^n x\,dx$, then show that

$$I_n + I_{n-2} = \frac{1}{n-1}$$

and deduce the value of $I_5$. 5

( Old Course )

*Full Marks* : 80

*Pass Marks* : 32

*Time* : 3 hours

1. State which of the following statements are true and which are false : 1×7=7

   *(a)* A function of the type $f(x, y) = 0$ is called implicit function.

   *(b)* If $x = \phi(a)$, $y = \psi(t)$, then

   $$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx}$$

   *(c)* The value of

   $$\int_0^{\pi/2} \sin^6 x \, dx$$

   is $\frac{5\pi}{32}$.

   *(d)* The union of two closed sets is not a closed set.

   *(e)* The set of all integers is countable.

   *(f)* Every bounded sequence has a limit point.

*(g)* According to d'Alembert's ratio test, $\lim_{n \to \infty} \frac{u_{n+1}}{u_n} = l < 1$ means that the series $\Sigma u_n$ is convergent.

**2.** *(a)* If $x^3 + y^3 - 3axy = 0$, then show that

$$\frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$$

5

*(b)* If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$$

6

*Or*

*(c)* Show that

*(i)* $D^n(x^n) = n!$

*(ii)* $D^n\left(\frac{1}{x+a}\right) = \frac{(-1)^n n!}{(x+a)^{n+1}}$

3+3=6

*(d)* Define maxima and minima of a function. Find for what values of $x$, the expression $f(x) = 2x^3 - 15x^2 + 36x + 10$ is maximum and minimum respectively, and hence find the maximum and minimum values. 2+5=7

*Or*

*(e)* State Leibnitz theorem for the $n$th derivative of the product of two functions. Using the theorem or otherwise, show that $x^2y_2 + xy_1 + y = 0$ for $y = a\cos(\log x) + b\sin(\log x)$. 2+5=7

**3.** *(a)* Show that

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

Using this property or otherwise, prove that

$$\int_0^{\pi/2} \frac{\cos x - \sin x}{1+\sin x \cos x}\,dx = 0$$

2+4=6

*(b)* Prove that

$$\int_0^{\pi/2} \sin^{2m} x\,dx = \frac{(2m)!}{\{2^m m!\}^2}\frac{\pi}{2}$$

5

*Or*

*(c)* Find : 5

$$\int_1^2 \int_0^x \frac{dx\,dy}{x^2+y^2}$$

*(d)* If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$ and $z = r\cos\theta$, then show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta$$

5

*(e)* Find Laplace transform of the function $(t+2)^2 e^t$. 3

**4.** *(a)* Define Cartesian product of two sets. Find $A \times B$ if $A = \{x | x = 1, 2\}$, $B = \{y | y = x + 2\}$. 1+3=4

*(b)* Show that a countable union of countable sets is countable. 4

*Or*

*(c)* Prove that a set is closed iff its complement is open. 4

*(d)* What are infimum and supremum of a set? Find the infimum and supremum of the sets $S_1 = \{2, 4, 6, 8\}$ and $S_2 = \left\{\frac{1}{n}, n \in N\right\}$. 5

*Or*

*(e)* If $S, T$ are subsets of real numbers, then—

*(i)* show that $(S \cap T)' \subseteq S' \cap T'$;

*(ii)* give an example to show that $(S \cap T)'$ and $(S' \cap T')$ may not be equal. 2½+2½=5

*(f)* Define a field' stating clearly its properties. 5

*Or*

*(g)* Define a set function. For the finitely additive set function $f$ defined on the field $F$, prove that

$$f(A \cup B) + f(A \cap B) = f(A) + f(B), \forall \ A, B \in F$$

1+4=5

**5.** *(a)* Define convergent, divergent and oscillatory series. Give an example of a series used in statistical analysis, which is convergent. 3+1=4

*(b)* What is a monotonic sequence? If $x_n = \dfrac{3n-1}{n+2}$, then prove that the sequence $\{x_n\}$ is monotone increasing and bounded. 5

*Or*

*(c)* Show that the sequence $\{S_n\}$, defined by the recursion formula $S_{n+1} = \sqrt{3S_n}$, $S_1 = 1$ converges to 3. 5

*(d)* Define Cauchy's root test and hence test for the convergence of the series where the general term is $\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$. 2+2=4

*Or*

*(e)* Show that the series

$$1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\cdots$$

is convergent. 4

*(f)* Show that the series

$$\Sigma\frac{3.6.9.\cdots.3n}{7.10.13.\cdots.(3n+4)}x^n, x>0$$

converges for $x\leq 1$ and diverges for $x>1$. 5

*Or*

*(g)* Prove that every absolutely convergent series is convergent. Show that for any fixed values of $x$, the series $\sum_{n=1}^{\infty}\frac{\sin nx}{x^2}$ is convergent. 3+2=5

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