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2 SEM TDC STS M 1 (N/O)

2017

(May)

STATISTICS

(Major)

Course : 201

(Mathematics for Statistics—I)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 48

Pass Marks : 14

Time : 2 hours

1. Choose the correct answer : 1×6=6

(a) If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$, then A and B are

(i) $A = \{a, x\}$, $B = \{b, y\}$

(ii) $A = \{a, a\}$, $B = \{y, y\}$

(iii) $A = \{a, y\}$, $B = \{b, x\}$

(iv) $A = \{a, b\}$, $B = \{x, y\}$

(b) One of the conditions for convergence of alternating series

$$\sum_{n=1}^{\infty} (-1)^{n-1} u_n$$

by Leibnitz test is that

(i) $\underset{n \to \infty}{Lt} u_n = k, \ k > 0$

(ii) $\underset{n \to \infty}{Lt} u_n = 1$

(iii) $\underset{n \to \infty}{Lt} u_n = 0$

(iv) $\underset{n \to \infty}{Lt} u_n \neq 0$

(c) The nth derivative of the function $f(x) = e^{5x}$ is

(i) e^{5x}

(ii) $5^n e^{5x}$

(iii) e^{5nx}

(iv) $x^n e^{5x}$

(d) If $x = r \cos \theta$, where r, x, θ are variables, then $\frac{\partial x}{\partial r}$ is

(i) $\cos \theta$

(ii) $r \sin \theta$

(iii) $\sin \theta$

(iv) 0

(e) Which one of the following is not true for the function $f(x)$?

(i) $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

(ii) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(iii) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

(iv) $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$, $f(x)$ is odd

(f) The value of $\int_0^{\pi/2} \cos^5 x\,dx$ is

(i) $\frac{15}{8}$

(ii) $\frac{8}{15}\pi$

(iii) $\frac{8}{15}$

(iv) $\frac{8}{15}\frac{\pi}{2}$

2. (a) Define partition of sets. Let $A = \{2\}$, $B = \{1, 3\}$, $C = \{4, 6\}$ and $D = \{1, 2, 3, 4, 5, 6\}$. Is $\{A, B, C\}$ a partition of D? Justify. 3

(b) Define countable set, equivalent sets and union of sets. Give one example in each case. 3

3. *(a)* Define limit point of a sequence. Prove that a sequence cannot converge to more than one limit. 1+5=6

Or

(b) Define divergent sequence. Give an example. Show that the sequence $\{S_n\}$ where

$$S_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{n+n} \quad \forall\, n \in N$$

is bounded and monotonic increasing. 2+4=6

4. *(a)* Give the d' Alembert's ratio test. Using the test or otherwise, prove that the series

$$\sum_{x=0}^{\infty} \frac{xe^{-\lambda}\lambda^x}{\lfloor x}, \quad \lambda > 0$$

is convergent. 2+4=6

Or

(b) When is a series said to be convergent? Prove that every absolutely convergent series is convergent. 1+5=6

5. (a) Differentiate $\sin x$ w.r.t. x^2. 2

(b) Show that $f(x) = e^x$ does not have maxima or minima. 2

6. Answer any *two* : 5×2=10

(a) Define increasing and decreasing functions. Find the intervals in which the function

$$f(x) = [x(x-2)]^2$$

is an increasing function. 1+4=5

(b) If $u = \log(x^2 + y^2)$, then prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

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(c) A random sample of size 400 is to be collected from two villages A and B for an economic survey. The cost of collecting m units from A and n units from B is given by the cost function

$$f(m, n) = 3m^2 + mn + 2n^2 + 250$$

Use the method of Lagrange's multiplier to determine m and n in such a way that the cost of collecting data is minimum. 5

7. (a) Evaluate the following with the help of integration : 5

$$\underset{n\to\infty}{\text{Lt}}\left[\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+\frac{n}{(n+3)^2}+\ldots+\frac{n}{(n+n)^2}\right]$$

Or

(b) Obtain a reduction formula for $\int \sin^n x\,dx$ and hence evaluate $\int \sin^3 x\,dx$. 4+1=5

8. (a) If $x=\frac{u}{u+v}$, $y=u+v$, find $J\left(\frac{u,\ v}{x,\ y}\right)$. 5

Or

(b) Evaluate : 5

$$\int_{y=2}^{3}\int_{x=0}^{y-1}\frac{dy\,dx}{y}$$

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. Choose the correct answer : 1×8=8

(a) If

$$u(x, y) = 2x^2 + 3x^2y$$

then $\frac{\partial u}{\partial y}$ is given by

(i) $3x^2$

(ii) $2x^2 + 6x$

(iii) $5x^2$

(iv) $4x + 6xy$

(b) If $f(c)$ is minimum of $f(x)$, then

(i) $f'(c) = 0$

(ii) $f'(c) \neq 0$

(iii) $f'(c) > 0$, always

(iv) $f'(c) < 0$, always

(c) If $F(t) = 1$, then $f(s) = L\{F(t)\}$ is

(i) $\frac{1}{s}$

(ii) $\frac{1}{s-1}$

(iii) $\frac{1}{s+1}$

(iv) $\frac{1}{s^2}$

(d) The value of $\int_0^{\pi/2} \sin^4 x\, dx$ is

(i) $\frac{3\pi}{16}$

(ii) $\frac{16\pi}{3}$

(iii) $\frac{3\pi}{8}$

(iv) None of the above

(e) If $A = \{1, 2\}$, then power set $P(A)$ of A is

(i) $\{\{1\}, \{2\}, \{1, 2\}\}$

(ii) $\{\phi, \{1\}, \{2\}, \{1, 2\}\}$

(iii) $\{\{1\}, \{2\}\}$

(iv) $\{\{\phi\}, \{1, 2\}\}$

(f) Which one of the following is not true for the sets A and B?

(i) $A \cap A = \phi$

(ii) $A \cap A = A$

(iii) $A \cap B = B \cap A$

(iv) $A \cup B = B \cup A$

(g) A sequence is said to be monotonic if it is

(i) strictly increasing

(ii) strictly decreasing

(iii) increasing only

(iv) either monotonic increasing or monotonic decreasing

(h) If $l = \underset{n \to \infty}{\text{Lt}} u_n^{1/n}$, then Cauchy's root test fails if

(i) $l > 1$

(ii) $l = 1$

(iii) $l < 1$

(iv) $l = 0$

2. (a) Define maxima and minima of a function. 2

(b) Differentiate x^5 w.r.t. x^2. 2

3. Answer any *two* : 7×2=14

(a) State and prove Leibnitz theorem for nth derivative of product of two functions. Also find $\frac{dy}{dx}$ if

$$x^3 - xy^2 + 3y^2 + 4 = 0$$ 5+2=7

(b) Define monotonically increasing and decreasing functions of x. Find the maximum value of the function

$$f(x) = 41 - 72x - 18x^2$$ 2+5=7

(c) What is Lagrange's undetermined multiplier λ? If $z = \frac{x^2y^2}{x+y}$, then show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3z$. 2+5=7

4. Answer any *three* : 6×3=18

(a) If $u = \frac{x_2x_3}{x_1}$, $v = \frac{x_1x_3}{x_2}$, $w = \frac{x_1x_2}{x_3}$, find

$$J\left(\frac{u,\ v,\ w}{x,\ y,\ z}\right)$$ 6

(b) Define Laplace Transform (LT) of a function $F(t)$ and give one application of LT in statistics. Find LT of the function

$$F(t) = 2t^n + 26e^{2t}$$ 2+4=6

(c) Evaluate : 6

$$\int_{x=0}^{1}\int_{y=x}^{\sqrt{x}} (x^2 + y^2)\, dx\, dy$$

(d) Obtain a reduction formula for

$$\int_0^{\pi/2} \cos^n x\, dx$$

and hence evaluate

$$\int_0^{\pi/2} \cos^3 x\, dx$$ 5+1=6

(e) Using the properties of definite integrals, show that

$$\int_0^{\pi} \frac{x\, dx}{1+\sin x} = \pi$$ 6

5. (a) What is partition of sets? Write down two important properties of partition of sets. 2+2=4

(b) Define union and intersection of sets. Give example. If $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6\}$, find the difference of sets $A - B$. 3+1=4

(c) Define equivalent sets. Give example. What is limit point of a sequence of sets? 2+2=4

6. (a) Prove that the Cartesian product of two countable sets is countable. 6

Or

(*b*) If S and T are subsets of real numbers, then show that : 3+3=6

(*i*) $S \subseteq T \Rightarrow S' \subseteq T'$

(*ii*) $(S \cap T)' \subseteq S' \cap T'$

7. (*a*) Define a sequence of real numbers. Give example. 2

(*b*) Define convergent and divergent sequences. 2

8. Answer any *two* : 7×2=14

(*a*) Prove that every convergent sequence is bounded. 7

(*b*) Define Leibnitz test for alternating series. Show that the series

$$\frac{1}{1^p} - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \dots$$

converges for $p > 0$. 2+5=7

(*c*) For any sequence $\{a_n\}$, show that

$$\inf a_n \leq \underline{\lim}\, a_n \leq \overline{\lim}\, a_n \leq \sup a_n$$ 7

(*d*) Show that the sequence $\{a_n\}$, where

$$a_n = \left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \dots + \frac{1}{(2n)^2}\right]$$

converges to 0. 7

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