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2 SEM TDC STS M 1 (N/O)

2018

(May)

STATISTICS

(Major)

Course : 201

(Mathematics for Statistics—I)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 48

Pass Marks : 14

Time : 2 hours

1. Choose the correct answer : 1×6=6

(a) If $A = \{a, b, c, d, e\}$, $B = \{a, c\}$, $C = \{b, d, e\}$, $D = \{b\}$, $E = \{d, e\}$, then which one of the following is not a partition of the set A?

(i) $\{B, D, E\}$

(ii) $\{B, C, D, E\}$

(iii) $\{B, D\}$

(iv) $\{A\}$

(b) Which of the following is not equivalent to $A \subset B$?

(i) $A - B = \phi$

(ii) $A \cap B = A$

(iii) $A \cup B = B$

(iv) None of the above

(c) By Cauchy's root test $\text{Lt}(u_n)^{\frac{1}{n}} > 1$ means a positive term series $\sum u_n$ is

(i) convergent

(ii) divergent

(iii) oscillatory

(iv) convergent and to 1 only

(d) If $S_{n+1} \geq S_n$, then the sequence $\{S_n\}$ is

(i) monotonic increasing

(ii) strictly increasing

(iii) monotonic decreasing

(iv) oscillatory

(e) The third derivative of e^{-2x} is

(i) $3e^{-2x}$

(ii) $-8e^{-2x}$

(iii) $-8e^{-8x}$

(iv) $-8e^{-6x}$

(f) $\int_a^b f\,dx = \int_a^c f\,dx + \int_c^b f\,dx$

(i) for any c

(ii) for $a < c < b$

(iii) c is exterior to the interval (a, b)

(iv) for all $c \neq 0$

2. *(a)* Define a set function. Find $A \times (B \cap C)$, where $A = \{a, b, c\}$, $B = \{c, d\}$, $C = \{d, e, f\}$. 3

(b) Define countable set, equivalence of sets and union of sets. Give one example in each case. 3

3. *(a)* Prove that the Cartesian product of two countable sets is countable. 6

Or

(b) Define a bounded sequence. If $\{a_n\}$ is a bounded sequence such that $a_n > 0$ for all $n \in N$, then show that

$$\underline{\lim}\left(\frac{1}{a_n}\right) = \frac{1}{\overline{\lim}\, a_n}, \text{ if } \overline{\lim}\, a_n > 0$$

1+5=6

4. *(a)* Define Cauchy's root test and hence test for the convergence of the series where the general term is

$$\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$$

2+4=6

Or

(b) What is a monotonic sequence? Show that the sequence $\{S_n\}$ is monotonic increasing, where

$$S_n = \frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n}, \ \forall\, n \in N$$

6

5. *(a)* Differentiate x^5 w.r.t. x^2. 2

(b) If $x = r\cos\theta$, $y = r\sin\theta$, then show that

$$r = \sqrt{x^2+y^2},\ \theta = \tan^{-1}\frac{y}{x}$$

2

6. Answer any *two* : 5×2=10

(a) State Leibnitz theorem for the *n*th derivative of the product of two functions and hence find *n*th derivative of $y = x^2 e^{ax}$.

(b) If $z = \frac{x^2 y^2}{x+y}$, then prove that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3z$$

(c) Using Lagrange's method of undetermined multiplier, find x and y in such way that $x + y = 100$ and the product xy becomes maximum.

7. (a) Obtain a reduction formula for $\int \tan^n x dx$ and hence evaluate $\int \tan^5 x dx$. 4+1=5

Or

(b) If $f(x) = f(a+x)$, then prove that

$$\int_0^{na} f(x) dx = n\int_0^a f(x) dx$$ 5

8. (a) Evaluate : 5

$$\int_1^2 \int_0^x \frac{dx\,dy}{x^2 + y^2}$$

Or

(b) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$, then show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta$$ 5

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. Choose the correct answer : 1×8=8

(a) The function $f(x) = x^2,\ x \in (0, \infty)$ is

(i) strictly decreasing

(ii) strictly increasing

(iii) non-increasing

(iv) non-decreasing

(b) The second derivative of the function $y = e^{ax}$ is

(i) ae^{ax}

(ii) a^2e^{ax}

(iii) $2a^2e^{ax}$

(iv) e^{ax} / a^2

(c) $\int_a^b f(x)dx = \int_a^{c_1} f(x)dx + \int_{c_1}^{c_2} f(x)dx + \int_{c_2}^b f(x)dx$

(i) for any c_1, c_2

(ii) $a < c_1 < c_2 < b$

(iii) for all $c_2 > c_1$

(iv) for all $c_1 \neq 0,\ c_2 \neq 0$

(d) The value of $\int_0^{\pi/2} \sin^6 x\, dx$ is

(i) $\frac{5\pi}{64}$

(ii) $\frac{5\pi}{32}$

(iii) $\frac{5}{32}$

(iv) $\frac{32\pi}{5}$

(e) Which one of the following is not true for equivalence of sets A, B and C?

(i) if $A \sim B$, then $A \sim C$

(ii) $A \sim A,\ B \sim B,\ C \sim C$

(iii) if $A \sim B,\ B \sim C$, then $A \sim C$

(iv) if $A \sim B$, then $B \sim A$

(f) If $A = \{a, b, c\}$, $B = \{b, d, e\}$, then

(i) $A \cup B = \{a, b, c, d, e\}$

(ii) $A \cup B = \{a, b, c, b, d, e\}$

(iii) $A \cap B = \{a, c, d, e\}$

(iv) $A \cap B = \phi$

(g) If the series $\sum u_n$ is absolutely convergent, then $\sum u_n$

(i) is always convergent

(ii) is always divergent

(iii) may or may not be convergent

(iv) is convergent under certain conditions

(h) If

$$\underset{n \to \infty}{\text{Lt}} (u_{n+1} / u_n) = l$$

then the D'Alembert's ratio test fails if

(i) $l = 1$

(ii) $l = 0$

(iii) $l < 1$

(iv) $l > 1$

2. (a) Differentiate $\sin x$ w.r.t. x^2. 3

(b) If $f(x, y) = 2x^2 - 3xy + 6x^3y$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. 2

(c) Show that $D^n(x^n) = \lfloor\underline{n}$ 3

3. Answer any *two* : 5×2=10

(a) If $x^3 + y^3 - 3axy = 0$, then show that

$$\frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$$

5

(b) Define minima and maxima of a function $f(x)$. Find for what value(s) of x the function $f(x) = 41 - 72x - 18x^2$ attains its maximum. 5

(c) If $y = a\cos(\log x) + b\sin(\log x)$, then show that $x^2y_2 + xy_1 + y = 0$. 5

4. (a) Show that

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

3

(b) Define Laplace transform of a function $F(t)$ and give two of its applications in statistics. 3

5. Answer any *two* : 6×2=12

(a) Evaluate : 6

$$\int_0^3 \int_1^2 xy(1+x+y)\, dx\, dy$$

(b) If $u = x/(x+y)$, $v = x+y$, then find

$$J\left(\frac{x, y}{u, v}\right)$$

6

(c) Using the properties of definite integrals, show that

$$\int_0^{\pi/2} \frac{\sin x - \cos x}{\sin x + \cos x}\, dx = 0$$

6

(d) Obtain a reduction formula for

$$\int \sin^n x\, dx$$

Find Laplace transform of the function

$$\frac{1}{2}(e^{at} + 3)$$

4+2=6

6. (a) Define set, power set and countable set. Give example for each. 3

(b) Define a field and a set function. 3

7. Answer any *two* : 6×2=12

(a) Define limit point of a set. If S and T are subsets of real numbers, then show that

$$(S \cup T)' = S' \cup T'$$ 1+5=6

(b) When is a set said to be closed? Prove that a set is closed iff its complement is open. 1+5=6

(c) Define Cartesian product of two sets. Show that a countable union of countable sets is countable. 1+5=6

8. *(a)* Define bounded sequence and convergence of sequence. 4

(b) Define positive term series and give the comparison test of first type for such series. 4

9. Answer any *two* : 5×2=10

(a) Prove that every bounded sequence has a limit point. 5

(b) Show that the sequence $\{S_n\}$, where

$$S_n = 1 + \frac{1}{\underline{|2}} + \frac{1}{\underline{|3}} + \cdots + \frac{1}{\underline{|n}}, \quad \forall \; n \in N$$

is convergent. 5

(c) Test the convergence of the series

$$\sum \frac{n^2-1}{n^2+1} x^n, \quad x > 0$$

(d) Show that the series

$$\sum_{x=0}^{\infty} \frac{e^{-\lambda x} \lambda^x}{\lfloor x}, \quad \lambda > 0$$

is convergent. What is the importance of the result in statistics? 4+1=

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