2 SEM TDC STS M 1 (N/O)

2019

(May)

STATISTICS

(Major)

Course : 201

(Mathematics for Statistics—I)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 48

Pass Marks : 14

Time : 2 hours

1. Choose the correct answer from the following alternatives : 1×5=5

(a) The set $S = \{\frac{1}{n}, n \in N\}$ is bounded, where (N is the set of natural numbers)

(i) the supremum 1 belongs to S and infimum 0 does not

(ii) the supremum 1 and infimum 0 both do not belong to S

(iii) the supremum n belongs to S and infimum 1 does not

(iv) None of the above

(b) For a non-empty class of sets a ring $\mathscr{R}$, if set $A \in \mathscr{R}$ and $B \in \mathscr{R}$, then

(i) $A \cup B \in \mathscr{R}$

(ii) $A - B \in \mathscr{R}$

(iii) Both (i) and (ii) are true

(iv) Neither (i) nor (ii) is true

(c) If $y = 2^{2^x}$, then $\frac{dy}{dx} =$

(i) $y(\log_{10} 2)^2$

(ii) $y(\log_e 2)^2$

(iii) $y2^x(\log_e 2)^2$

(iv) $y\log_e 2$

(d) $\lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$, if exists, is called the partial derivative of f with respect to

(i) x at (a, b)

(ii) x at (x, y)

(iii) y at (a, b)

(iv) y at (x, y)

(e) $\int_0^a f(x)\,dx =$

(i) $\int_0^a f(-x)\,dx$

(ii) $\int_0^a f(a+x)\,dx$

(iii) $\int_0^a f(a-x)\,dx$

(iv) $\int_0^a f(2a-x)\,dx$

2. Answer the following in brief : 2×5=10

(a) Construct the smallest field from a partitioned class of sets $\{A_1, A_2, A_3\}$.

(b) If $\{A_n\}$ is an arbitrary sequence of sets, then prove that—

(i) $(\lim\sup A_n)^C = \lim\inf A_n^C$;

(ii) $(\lim\inf A_n)^C = \lim\sup A_n^C$.

(c) If $y = a\cos(\log x) + b\sin(\log x)$, then show that

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

(d) Show that the maximum value of $\left(\frac{1}{x}\right)^x$ is $(e)^{1/e}$.

(e) Evaluate :

$$\int_0^{\pi}\int_0^{\pi} \sin y \, dy dx$$

3. (a) Define a countable set with examples. Prove that the set of rational numbers in [0, 1] is countable. 2+4=6

Or

(b) What is a monotonic sequence? If $\{S_n\}$ be a sequence such that

$$S_{n+1} = 2 - \frac{1}{S_n}, \; n \geq 1 \quad \text{and} \quad S_1 = \frac{3}{2}$$

then show that the sequence $\{S_n\}$ is bounded and monotonic and converges to 1. 2+4=6

4. (a) What do you mean by an infinite series? Prove that a necessary condition for convergence of an infinite series $\sum_{n=1}^{\infty} u_n$ is that $\lim_{n \to \infty} u_n = 0$. 2+4=6

Or

(b) What is D'Alembert's ratio test? Test for convergence of the series

$$\sum \frac{n^2 - 1}{n^2 + 1} \cdot x^n, \; x > 0$$ 2+4=6

5. (a) Define increasing and decreasing functions. Find the intervals in which the function $f(x)=[x(x-2)]^2$ is an increasing function. 1+5=6

Or

(b) (i) Find $\frac{dy}{dx}$ when $x=a\cos^3 t$ and $y=a\sin^3 t$. 3

(ii) If $x^y=e^{x-y}$, then prove that

$$\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$$ 3

6. (a) State and prove Leibnitz theorem. 5

Or

(b) Using Lagrange's method of undetermined multiplier, find x and y in such a way that $x+y=100$ and the product xy becomes maximum. 5

7. Answer any *two* of the following : 5×2=10

(a) Write down the reduction formula for $\int_0^{\pi/2}\sin^n x\,dx$, n is a positive integer and hence evaluate $\int_0^{\pi/2}\sin^6 x\,dx$.

(b) If $x = \frac{u}{u+v}$, $y = u + v$, then find $J\left(\frac{u, v}{x, y}\right)$.

(c) Evaluate :

$$\int_0^a \int_0^a \int_0^a (x^2 + y^2 + z^2)\, dxdydz$$

(d) Evaluate the following with the help of integration :

$$\lim_{n \to \infty} \frac{[(n+1)(n+2) \cdots (n+n)]^{1/n}}{n}$$

(Old Course)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

1. Choose the correct answer from the following alternatives :

1×8=8

(a) If $f(x)$ be a maximum or a minimum at $x = c$ and if $f'(c)$ exists, then

(i) $f'(c) = 0$

(ii) $f''(c) = 0$

(iii) $f'(c)$ is negative

(iv) $f'(c)$ is positive

(b) The third derivative of the function $y = e^{ax}$ is

(i) $a^3 e^{ax}$

(ii) $6e^{ax}$

(iii) $3a^3 e^{ax}$

(iv) e^{ax} / a^3

(c) The value of $\int_a^b f(t)\,dt$ is always same as the value of

(i) $-\int_b^a f(x)\,dx$

(ii) $\int_{-a}^{-b} f(t)\,dt$

(iii) $-\int_b^a f(-t)\,dt$

(iv) $\int_a^b f(-t)\,dt$

(d) The value of $\int_0^{\pi/2} \sin^2 dx$ is

(i) $\pi / 2$

(ii) $\pi / 4$

(iii) π

(iv) 2π

(e) Any non-empty subset of real numbers which is bounded below has

(i) infimum

(ii) both infimum and supremum

(iii) supremum

(iv) neither infimum nor supremum

(f) Which of the following is not equivalent to $A \subset B$?

(i) $A - B = \phi$

(ii) $A \cap B = A$

(iii) $A \cup B = B$

(iv) None of the above

(g) If $A = \{1, 2\}$, then the power set $P(A)$ of A is

(i) $\{\{1\}, \{2\}, \{1, 2\}\}$

(ii) $\{\phi, \{1, 2\}, \{1\}, \{2\}\}$

(iii) $\{\{1\}, \{2\}\}$

(iv) $\{\{\phi\}, \{1, 2\}\}$

(h) If $S_{n+1} \geq S_n$, then the sequence $\{S_n\}$ is

(i) monotonic increasing

(ii) strictly increasing

(iii) monotonic decreasing

(iv) oscillatory

(Continued)

2. Answer the following in brief : 4×4=16

(a) If $z = \frac{x^2y^2}{x+y}$, then prove that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3z$$

(b) Evaluate :

$$\int_{x=0}^{1}\int_{y=x}^{\sqrt{x}} (x^2 + y^2)\,dxdy$$

(c) Show that the set of rational numbers in (0, 1) is uncountable.

(d) Show that the sequence $\{S_n\}$, where

$$S_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}, \quad \forall\, n \in N,$$

is convergent.

3. Answer any *two* of the following : 7×2=14

(a) Show that the function

$$f(x) = 4x^3 - 6x^2 - 72x + 30$$

is strictly increasing in the intervals $(-\infty, -2)$ and $(3, \infty)$, and strictly decreasing in $(-2, 3)$.

(b) State Leibnitz theorem for the *n*th derivative of the product of two functions and hence find the *n*th derivative of $y = x^2e^{ax}$.

(c) Define maxima and minima of a function. Find for what values of x, the expression $f(x) = 2x^3 - 15x^2 + 36x + 10$ is maximum and minimum respectively, and hence find the maximum and minimum values.

(d) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$$

4. Answer any *two* of the following : 7×2=14

(a) Obtain a reduction formula for $\int \tan^n x\,dx$ and hence evaluate $\int \tan^5 x\,dx$. 4+3=7

(b) Using the properties of definite integrals, show that

$$\int_0^{\pi} \frac{x\,dx}{1+\sin x} = \pi$$ 7

(c) Define Jacobian of transformation. If

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

then show that $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta$. 2+5=7

(d) Define Laplace transform of a function $F(t)$ and mention two of its important properties. Find the Laplace transform of the function

$$F(t) = \frac{e^{-at} - 1}{a}$$ 2+2+3=7

5. Answer any *two* of the following : 7×2=14

(a) What do you mean by class of sets? Define ring, semi-ring and field. Prove that a class of sets closed under complementation and finite unions is a field. 1+3+3=7

(b) Define a countable set. Prove that the set of all rational numbers is countable. 1+6=7

(c) What is partition of sets? Write down two important properties of partition of sets. Construct the smallest field from a partitioned class of sets $\{A_1, A_2, A_3\}$. 1+2+4=7

6. Answer any *two* of the following : 7×2=14

(a) What is bounded sequence? Prove that every convergent sequence is bounded and has a unique limit. 2+2½+2½=7 *(b)* Define D'Alembert's ratio test. By virtue of D'Alembert's ratio test, test whether the series $\sum \frac{n^2 - 1}{n^2 + 1} \cdot x^n$, $x > 0$ is convergent or divergent. 3+4=7

(c) Show that a positive term series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent iff $p > 1$. Define Cauchy's root test and mention about the decision taken when the test fails. 4+3=7

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