Total No. of Printed Pages—12

2 SEM TDC STS M 1 (N/O)

2020

STATISTICS

(Major)

Course : 201

(Mathematics for Statistics—I)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 48

Pass Marks : 14

Time : 2 hours

1. Choose the correct answer from the following alternatives : 1×5=5

(a) If $S_{n+1} \geq S_n$, then the sequence $\{S_n\}$ is

(i) monotonic

(ii) strictly increasing

(iii) monotonic decreasing

(iv) oscillatory

(b) If $l = \underset{n \to \infty}{\text{Lt}}\ u_n^{\frac{1}{n}}$, then Cauchy's root test fails if

(i) $l > 1$ (ii) $l = 1$

(iii) $l < 1$ (iv) $l = 0$

(c) If $f(x)$ be a maximum or minimum at $x = c$ and if $f'(c)$ exists, then

(i) $f'(c) = 0$

(ii) $f''(c) = 0$

(iii) $f'(c)$ is negative

(iv) $f'(c)$ is positive

(d) If $x = r\cos\theta$, where r, x, θ are variables, then $\frac{\partial x}{\partial r}$ is

(i) $\cos\theta$ (ii) $r\sin\theta$

(iii) $\sin\theta$ (iv) 0

(e) The value of $\int_0^{\pi/2} \sin^4 x\, dx$ is

(i) $\frac{3\pi}{16}$

(ii) $\frac{16\pi}{3}$

(iii) $\frac{3\pi}{8}$

(iv) None of the above

2. Answer the following in brief : 2×5=10

(a) Define countable set and equivalent sets. Give one example in each case.

(b) What is divergent sequence? Give an example of it.

(c) Show that the function $f(x) = x^2 - 6x$ is increasing for $x > 3$.

(d) Show that $f(x) = e^x$ does not have maxima or minima.

(e) Prove that

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

3. (a) Define limit point of a sequence. Prove that a sequence can not converge to more than one limit. 1+5=6

Or

(b) Define a bounded sequence. If $\{a_n\}$ is a bounded sequence such that $a_n > 0$ for all $n \in N$, then show that

$$\underline{\lim}\left(\frac{1}{a_n}\right) = \frac{1}{\overline{\lim}\, a_n}, \quad \text{if } \overline{\lim}\, a_n > 0$$

1+5=6

4. (a) When is a series said to be convergent? Prove that every absolutely convergent series is convergent. 1+5=6

Or

(b) Define Cauchy's root test. Show that a positive term series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

is convergent iff $p > 1$. 2+4=6

5. (a) Define maxima and minima of a function. Find for what values of x, the expression

$$f(x) = 2x^3 - 15x^2 + 36x + 10$$

is maximum and minimum respectively and hence find the maximum and minimum values. 1+5=6

Or

(b) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$$

6

6. (a) If $\sin y = x \sin(a+y)$, then prove that

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

5

Or

(b) State and prove Leibnitz theorem. 5

7. Answer any *two* of the following : 5×2=10

(a) Obtain a reduction formula for

$$\int_0^{\pi/2} \cos^n x\, dx$$

and hence evaluate

$$\int_0^{\pi/2} \cos^3 x\, dx$$

(b) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$, then show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta$$

(c) Evaluate :

$$\int_{x=0}^{1} \int_{y=x}^{\sqrt{x}} (x^2 + y^2)\, dx\, dy$$

(d) Using the properties of definite integrals, show that

$$\int_0^{\pi} \frac{x\, dx}{1 + \sin x} = \pi$$

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. Choose the correct answer from the following alternatives in each question : 1×8=8

(a) The function $f(x) = |x|$ at $x = 0$ is

(i) continuous and differentiable

(ii) continuous but not differentiable

(iii) not continuous but differentiable

(iv) neither continuous nor differentiable

(b) The function defined by $f(x) = x^{\frac{1}{x}}$ has maximum at

(i) 1 (ii) $e^{1/e}$

(iii) $\log_2 e$ (iv) 2

(c) The value of $\int_0^{\pi/2} \sin^4 x \, dx$ is

(i) $\frac{3\pi}{16}$

(ii) $\frac{3\pi}{8}$

(iii) $\frac{3}{16}$

(iv) None of the above

(d) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx,$ for all values of c, such that

(i) $a < c < b$

(ii) $c > 0$

(iii) $c > a$

(iv) None of the above

(e) With usual notation $A \cup (B \cap C)$ is equal to

(i) $(A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cap C)$

(iii) $(A \cap B) \cup (A \cap C)$

(iv) $A \cup (B \cup C)$

(f) If $A = \{1, 2\}$, then the power set $P(A)$ of A is

(i) $\{\{1\}, \{2\}, \{1, 2\}\}$

(ii) $\{\varnothing, \{1, 2\}, \{1\}, \{2\}\}$

(iii) $\{\{1\}, \{2\}\}$

(iv) $\{\{\varnothing\}, \{1, 2\}\}$

(g) By Cauchy's root test, $\lim (u_n)^{1/n} > 1$ means a positive term series Σu_n is

(i) convergent

(ii) divergent

(iii) oscillatory

(iv) convergent and to 1 only

(h) A convergent sequence is

(i) always bounded

(ii) bounded above only

(iii) bounded below only

(iv) neither bounded above nor bounded below

2. Answer the following : 4×4=16

(a) (i) Differentiate $\sin x$ w.r.t. x^2.

(ii) If $x = r\cos\theta$, $y = r\sin\theta$, then show that

$$r = \sqrt{x^2 + y^2},\ \theta = \tan^{-1}(y/x)$$

(b) If $L\{F(t)\} = f(s)$ is the Laplace transform (LT) of the function $F(t)$, then find

$$L\left\{t^n + \frac{1}{6}\right\} \text{ and } L\{e^{at}\}$$

(c) What are the infimum and supremum of a set? Find the infimum and supremum of the sets $S_1 = \{2, 4, 6, 8\}$ and $S_2 = \left\{\frac{1}{n},\ n \in N\right\}$.

(d) Define real sequence. Prove that a sequence converge to more than one limit.

3. Answer any *two* of the following : 7×2=14

(a) Define maxima and minima of a function. Find for what values of x, the expression

$$f(x) = 2x^3 - 15x^2 + 36x + 10$$

is maximum and minimum respectively, and hence find the maximum and minimum values. 2+5=7

(b) State Leibnitz theorem for the nth derivative of the product of two functions. Using the theorem or otherwise, show that $x^2y_2 + xy_1 + y = 0$ for $y = a\cos(\log x) + b\sin(\log x)$. 2+5=7

(c) (i) Find $\frac{dy}{dx}$, when $x = a\cos^2 t$ and $y = a\sin^3 t$. 3

(ii) If $x^y = e^{x-y}$, then prove that

$$\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$$

4

4. Answer any *two* of the following : 7×2=14

(a) Obtain the reduction formula for $\int_0^{\pi/2} \sin^n x\,dx$, where n is a positive integer and hence evaluate,

$$\int_0^{\pi/2} \sin^6 x\,dx$$ 7

(b) (i) Evaluate

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} (\sqrt{a^2-x^2-y^2})\,dy\,dx$$ 4

(ii) Prove that

$$\int_{-a}^{a} f(x) = 0; \qquad \text{if } f(-x) = -f(x)$$

$$= 2\int_0^a f(x)\,dx; \text{ if } f(-x) = f(x)$$ 3

(c) (i) If $u = x + y$ and $v = \frac{u}{x+y}$, then find

$$J\left(\frac{x,\ y}{u,\ v}\right)$$ 4

(ii) Define Laplace transform of a function $F(t)$ and mention two of its important properties. 3

(d) Evaluate

$$\lim_{n \to \infty}\left[\frac{n}{(n+1)^2} + \frac{n}{(n+2)^2} + \ldots + \frac{n}{(n+n)^2}\right]$$

with the help of definite integral. 7

5. Answer any *two* of the following : 7×2=14

(a) (i) Give one example of each of equivalent set, countable set and union of sets. 3

(ii) Define a set function. Find $A \times (B \cap C)$, where $A = \{a, b, c\}$, $B = \{c, d\}$, $C = \{d, e, f\}$. 2+2=4

(b) Define countable set with examples. Prove that the set of rational numbers in [0, 1] is countable. 2+5=7

(c) What is partition of sets? Write down two important properties of partition of sets. Construct the smallest field from a partitioned class of sets $\{A_1, A_2, A_3\}$. 1+2+4=7

(d) What is class of sets? Define ring, semi-ring and field. Show that a class of sets closed under complementation and finite unions is a field. 1+3+3=7

6. Answer any *two* of the following : 7×2=14

(a) Show that the sequence $\{a_n\}$ defined by

$$a_{n+1} = \frac{1}{2}\left(a_n + \frac{9}{a_n}\right), \quad n \geq 1, \quad a_1 > 0$$

is convergent and it converges to 3. 5+2=7 *(b)* Give a comparison test for positive term series Σu_n and Σv_n. Test the convergence of the series

$$\Sigma\,[(n^3+1)^{1/3}-1]$$

3+4=7

(c) Define a bounded sequence and show that the sequence $\{S_n\}$ is bounded, where

$$S_n = 1+\frac{1}{2!}+\frac{1}{3!}+\ldots+\frac{1}{n!}, \quad \forall\, n \in N$$

2+5=7

★★★