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3 SEM TDC GEMT (CBCS) GE 3 (A/B/C)

2023

(Nov/Dec)

MATHEMATICS

(Generic Elective)

Paper : GE-3

Full Marks : 80

Pass Marks : 32

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

Paper : GE-3A

(**Real Analysis**)

1. (a) Fill in the blank :

A set S is said to be _____ if it is finite
or denumerable. 1

(b) Is the set $E = \{2n : n \in \mathbb{N}\}$ denumer-
able? Justify. 3

(c) Prove that every subset of a countable
set is countable. 4

Or

Prove that union of a finite number of
countable sets is countable.

- (d) If $a \in \mathbb{R}$ and $a \neq 0$, then prove that $a^2 > 0$. 4
- (e) Let S be a non-empty subset of \mathbb{R} that is bounded above and let a be any number in \mathbb{R} . If $a+S = \{a+s : s \in S\}$, prove that $\sup(a+S) = a + \sup S$. 4

Or

- If $S = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$, prove that $\inf S = 0$.
- (f) State and prove the nested interval property. 4

2. (a) State True or False : 1

The range of a real sequence may be finite or infinite without ever being the null set.

- (b) Every convergent sequence is bounded. Is the converse true? Justify. 3
- (c) Write the limit point of the sequence $\{S_n\}$, where

$$S_n = (-1)^n \left(1 + \frac{1}{n} \right), n \in \mathbb{N}$$

Does the range set have limit points? 2+1=3

- (d) State and prove Bolzano-Weierstrass theorem for sequences. 4

Or

Prove that every bounded sequence with a unique limit point is convergent.

- (e) Prove that every Cauchy sequence is bounded. Is the converse true? 3+1=4

(f) Show that the sequence $\{S_n\}$, where

$$S_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

cannot converge.

3

Or

Show that the sequence $\{S_n\}$, where

$$S_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}$$

is convergent for all $n \in \mathbb{N}$.

(g) Is the sequence $\{n^2\}$ a Cauchy sequence? Justify.

2

3. (a) State the necessary condition for the convergence of an infinite series.

1

(b) State Cauchy's general principle of convergence for series.

1

(c) Prove that the positive term geometric series

$$1 + r + r^2 + r^3 + \dots$$

converges for $r < 1$ and diverges to $+\infty$ for $r \geq 1$.

5

Or

Discuss the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

- (d) Investigate the behaviour of the series whose n th term is

$$\sin\left(\frac{1}{n}\right)$$

3

Or

Write the three conditions of Leibnitz test.

- (e) Test the convergence of any *two* of the following : 5×2=10

(i) $\sum_{n=1}^{\infty} \{(n^3 + 1)^{1/3} - n\}$

(ii) $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + 1}$

(iii) $\frac{2^p}{1^q} + \frac{3^p}{2^q} + \frac{4^p}{3^q} + \dots$

(iv) $\frac{1}{1+2} + \frac{2}{1+2^2} + \frac{3}{1+2^3} + \dots$

4. (a) Test for uniform convergence of the sequence $\{f_n\}$, where

$$f_n(x) = \frac{nx}{1+n^2x^2}, \text{ for all real } x$$

5

Or

Show that the sequence $\{f_n\}$, where

$$f_n(x) = \frac{x}{1+nx^2}$$

is uniformly convergent on any closed interval I .

- (b) Let $\{f_n\}$ be a sequence of functions such that

$$\lim_{n \rightarrow \infty} f_n(x) = f(x), \quad x \in [a, b]$$

$$\text{and let } M_n = \sup_{x \in [a, b]} |f_n(x) - f(x)|$$

Prove that $f_n \rightarrow f$ uniformly on $[a, b]$ if and only if $M_n \rightarrow 0$ as $n \rightarrow \infty$. 5

Or

Let $\{f_n\}$ be a sequence of differentiable functions on $[a, b]$ such that it converges at least at one point $x_0 \in [a, b]$. If the sequence of differentials $\{f'_n\}$ converges uniformly to G on $[a, b]$, then prove that the sequence $\{f_n\}$ converges uniformly on $[a, b]$ to f and $f'(x) = G(x)$.

- (c) Give an example of a power series. 1

- (d) If a power series $\sum a_n x^n$ converges for $x = x_0$, then prove that it is absolutely convergent for every $x = x_1$ when $|x_1| < |x_0|$. 5

- (e) Determine the radius of convergence and the exact interval of convergence of any one of the following : 4

(i) $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots$

(ii) $1 + \frac{3}{5}x + \frac{3.5}{5.10}x^2 + \dots$

Paper : GE-3B

(Cryptography and Network Security)

1. (a) Write True or False : 1
Cryptography is used only for encoding the message.
- (b) Choose the correct option : 1
In public key encryption, the message is encrypted with the receiver's
(i) private key
(ii) key pair
(iii) symmetric key
(iv) public key
- (c) Choose the correct option : 1
In _____, same key is used for encryption and decryption.
(i) symmetric
(ii) asymmetric
(iii) public key
(iv) None of the above
- (d) Write True or False : 1
A message digest is also called hash.
- (e) What is cryptography? 2

(f) Explain how the private key symmetric encryption works. 10

Or

(g) Describe the RSA algorithm.

(h) Illustrate how digital signature works by giving an example. 8

Or

(i) Briefly explain the SHA-1 algorithm.

2. (a) Define two IP SEC protocols. 2

(b) Briefly explain VPN. 8

Or

(c) Explain denial of service attack.

(d) Define the following (any three) : $3 \times 3 = 9$

(i) IP spoofing

(ii) TCP session hijacking

(iii) Sequence guessing

(iv) Teardrop attack

(v) TCP sweeps

(e) Briefly explain how ICMP works. 5

3. (a) Briefly explain SNMP architecture. 6
- (b) What is firewall? Describe how firewall can be used to protect the network. 2+8=10

Or

- (c) Briefly explain the working of secure electronic transaction (SET). 10
- (d) Briefly explain the following (any four) : 4×4=16

- (i) Intrusion Detection System (IDS)
- (ii) Encapsulating Security Payload (ESP)
- (iii) SSL
- (iv) DSS
- (v) MAC
- (vi) Active attack and passive attack

Paper : GE-3C

(Information Security)

1. Answer any *five* of the following questions :

2×5=10

- (a) What is data integrity?
- (b) Write the differences between worm and virus in terms of information security.
- (c) What is a transpositional cipher?
- (d) What is an intrusion detection system?
- (e) What is a hash function?
- (f) Write the principles of security.
- (g) What is a trip-wise security mechanism?

2. (a) Compare and contrast substitution and transposition techniques.

3

(b) Briefly describe any *three* of the following :

4×3=12

- (i) Trojan horse
- (ii) Data availability
- (iii) MAC
- (iv) Buffer overflow

3. (a) Differentiate between symmetric and asymmetric encryptions. 5

(b) Explain Diffie-Hellman key exchange with both keys. Give example. 5+5=10

4. (a) Briefly explain the functionalities of data encryption standard (DES). 5

(b) Consider the following :

Plaintext : 'KEY'

SECRET KEY : "CRYPTOGRAPHY"

Compute the cipher text from the given plain text and key using hill cipher method. 5

Or

What are the properties that digital signature should have?

5. (a) Write the advantages and disadvantages of secret key encryption. 5

(b) In an RSA system, the public key of a given user is $e = 31$, $n = 3599$. What is the private key of this user? 5

6. (a) Generate public key and private key in case of RSA algorithm if two prime numbers given are 5 and 7. ($p = 5$ and $q = 7$) 5

(b) Briefly explain the system threats. 5

7. Explain any *two* of the following :

5×2=10

- (a) Auditing and logging
- (b) Public key signature
- (c) Program threats
- (d) Data integrity

★ ★ ★