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3 SEM TDC PHYH (CBCS) C 5

2023

(Nov/Dec)

PHYSICS

(Core)

Paper : C-5

(Mathematical Physics—II)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Choose the correct answer : 1×5=5

(a) The value of $\operatorname{erf}(\infty)$ is

(i) 1

(ii) 0

(iii) -1

(iv) None of the above

(b) The value of $\Gamma\left(-\frac{1}{2}\right)$ is

(i) $\sqrt{\pi}$

(ii) $\frac{-\pi}{2}$

(iii) $-2\sqrt{\pi}$

(iv) 0

(c) The value of $\int_{-1}^{+1} [P_3(x)]^2 dx$ is

(i) $\frac{2}{3}$

(ii) $\frac{2}{7}$

(iii) $\frac{1}{7}$

(iv) None of the above

(d) The value of Hermite polynomial $H_2(x)$ is

(i) $\frac{1}{2}(3x^2 - 1)$

(ii) $\frac{1}{3}(3x^2 - 1)$

(iii) $(4x^2 - 2)$

(iv) $\frac{1}{2}(4x^2 - 1)$

(e) According to the Parseval's formula for Fourier series, the integral $\int_{-c}^c [f(x)]^2 dx$ equals to

$$(i) \quad c \left\{ a_0^2 + \sum_{n=1}^{\infty} (a_n^2 + b_n^2) \right\}$$

$$(ii) \quad c \left\{ \frac{1}{2} a_0^2 + \sum_{n=1}^{\infty} (a_n^2 + b_n^2) \right\}$$

$$(iii) \quad c \left\{ a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2) \right\}$$

(iv) None of the above

2. (a) Describe Dirichlet's conditions for a Fourier series. 2

(b) Expand the function $f(x)$ in a Fourier series where $-\pi < x < \pi$ and $f(x)$ is given by

$$\begin{aligned} f(x) &= 0 \quad \text{when } -\pi < x \leq 0 \\ &= \frac{\pi x}{4} \quad \text{when } 0 < x \leq \pi \end{aligned}$$

Hence show that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots \quad 4+2=6$$

- (c) Obtain the complex form of the Fourier series for the function defined as follows :

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$$\begin{aligned} f(x) &= 0 \quad \text{when } -\pi < x \leq 0 \\ &= 1 \quad \text{when } 0 < x \leq \pi \end{aligned}$$

3. (a) Determine the nature of the point $x=0$

for $x \frac{d^2 y}{dx^2} + y \sin x = 0$. 1+2=3

- (b) Solve using Frobenius method (any one) : 5

(i) $x^2 y'' + (x + x^2) y' + (x - 9) y = 0$

(ii) $4xy'' + 2y' + y = 0$

- (c) Express the following in terms of Legendre polynomials :

3

$$4x^2 - 3x + 2$$

- (d) Prove that

$$\int_{-1}^{+1} x P_n(x) P_{n-1}(x) dx = \frac{2n}{4n^2 - 1} \quad 4$$

Or

Prove that

$$\int_{-\infty}^{\infty} e^{-x^2} H_m(x) H_n(x) dx = 0 \quad \text{for } m \neq n$$

4. Evaluate :

$$\int_0^{\infty} x^{n-1} e^{-h^2 x^2} dx = \frac{1}{2h^n} \Gamma\left(\frac{n}{2}\right) \quad 3$$

Or

Prove that

$$\int_0^{\pi/2} \sin^p \theta \cos^q \theta d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right) \Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}$$

5. Answer any *one* of the following : 6

(a) What is relative error? Describe with an

example. $R = \frac{4xy^2}{z^3}$ and errors in x, y, z

be 0.001, show that the maximum relative error at $x = y = z = 1$ is 0.006.

$$1+1+4=6$$

(b) Describe briefly the least square method of curve fitting. Fit a curve to the following data by the least square method :

$$2+4=6$$

x	1	2	3	4
y	1.7	1.8	2.3	3.2

6. (a) Solve any *two* of the following partial differential equations by method of separation of variables : 4×2=8

$$(i) \frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$$

$$(ii) \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \text{ if } u(x, 0) = \frac{1}{2}x(1-x)$$

$$(iii) \frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x \text{ given that :}$$

$$u = 0 \text{ when } t = 0$$

$$\frac{\partial u}{\partial t} = 0 \text{ when } x = 0$$

- (b) Find the solution of 2-D Laplace's equation in cylindrical polar co-ordinates. 5

Or

Find the vibration modes of a stretched string by solving the one-dimensional wave equation.

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