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3 SEM TDC STSH (CBCS) C 7 (N/O)

2023

(Nov/Dec)

STATISTICS

(Core)

Paper : C-7

(**Mathematical Analysis**)

*The figures in the margin indicate full marks
for the questions*

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the following alternatives : the
 $1 \times 6 = 6$

(a) The set N of natural numbers is.

(i) bounded above

(ii) bounded below

(iii) Both (i) and (ii)

(iv) None of the above

(Turn Over)

- (b) The series $\sum u_n$ of positive terms is convergent or divergent as

$$\lim_{n \rightarrow \infty} \frac{u_n}{u_{n+1}} > 1 \text{ or } < 1$$

Then the test is known as

- (i) comparison test
 - (ii) Raabe's test
 - (iii) Cauchy's condensation test
 - (iv) D'Alembert's test
- (c) The series $\sum u_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ is
- (i) absolute convergent
 - (ii) conditional convergent
 - (iii) Both (i) and (ii)
 - (iv) None of the above

- (d) A function f is said to be continuous from the left at c , if

(i) $\lim_{x \rightarrow c+0} f(x) = f(c)$

(ii) $\lim_{x \rightarrow c+0} f(x) = f(0)$

(iii) $\lim_{x \rightarrow c-0} f(x) = f(c)$

- (iv) None of the above

- (e) The n th divided difference of a polynomial of n th degree is
- (i) always zero
 - (ii) always equal to n
 - (iii) always constant
 - (iv) not defined
- (f) Lagrange's formula is useful for
- (i) interpolation
 - (ii) extrapolation
 - (iii) inverse interpolation
 - (iv) All of the above

2. Answer the following questions in brief: $2 \times 6 = 12$

- (a) Define derived set with examples.
- (b) State the Cauchy's general principle of convergence.
- (c) Define Raabe's test.
- (d) State the Taylor's theorem with the remainder in Cauchy's form.
- (e) State two properties of divided difference.
- (f) Define transcendental equation with examples.

3. Answer any *two* of the following questions :

$5 \times 2 = 10$

- (a) Show that the set of real numbers forms a complete ordered field.
- (b) Prove that a set is closed iff its complement is open.

(c) Show that

$$\lim_{n \rightarrow \infty} \left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+n}} \right] = 1$$

4. (a) (i) Define infinite series. Under what condition a geometric series is convergent? Show that the series

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots$$

is not convergent.

$$1+1+3=5$$

- (ii) Define Cauchy's n th root test. By virtue of D'Alembert's ratio test, test whether the series

$$\frac{1^2 \cdot 2^2}{1!} + \frac{2^2 \cdot 3^2}{2!} + \frac{3^2 \cdot 4^2}{3!} + \dots$$

is convergent or divergent.

$$2+4=6$$

Or

- (b) (i) Give the statement of Cauchy's condensation test. Test the convergence of the following series using Raabe's test :

$$1+4=5$$

$$\frac{\alpha}{\beta} + \frac{1+\alpha}{1+\beta} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \dots$$

- (ii) Show that every absolutely convergent series is convergent. Show that the series

$$\frac{\log 2}{2} + \frac{\log 3}{3} + \frac{\log 4}{4} + \dots \infty$$

is divergent.

$$3+3=6$$

5. (a) Define uniform continuity of a function. Find the values of a and b so that $f(x)$ may be differentiable at $x=1$, where

$$f(x) = \begin{cases} x^2 + 3x + a & ; \text{ if } x \leq 1 \\ bx + 2 & ; \text{ if } x > 1 \end{cases} \quad 2+5=7$$

Or

- (b) Give the statement of Rolle's theorem. Show that

$$\frac{v - \mu}{1 + v^2} < \tan^{-1} v - \tan^{-1} \mu < \frac{v - \mu}{1 + \mu^2},$$

if $0 < \mu < v$ and deduce that

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6} \quad 2+5=7$$

6. (a) Define the operator E and show that $E = 1 + \Delta$. Represent the function $f(x)$ given by

$$f(x) = 2x^4 - 12x^3 + 24x^2 - 30x + 9$$

and its successive differences in factorial notation. What do you mean by interpolation? Write the statement of Newton's forward interpolation formula.

$$1+2+3+3=9$$

Or

- (b) What do you mean by numerical integration? What are the basic conditions to apply Simpson's one-third rule? Solve $u_{x+1} - au_x = 0$; $a \neq 1$. Evaluate $\sqrt{12}$ by applying Newton's formula.

$$2+2+2+3=9$$

(6)

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. Choose the correct answer from the following : the
1×8=8

- (a) The set of natural numbers N is
- (i) bounded above
 - (ii) bounded below
 - (iii) Both (i) and (ii)
 - (iv) None of the above
- (b) If $S_{n+1} \geq S_n$, then the sequence $\{S_n\}$ is
- (i) monotonic increasing
 - (ii) strictly increasing
 - (iii) monotonic decreasing
 - (iv) oscillatory
- (c) The series $\sum u_n$ of positive terms is convergent or divergent as

$$\lim_{n \rightarrow \infty} \frac{u_n}{u_{n+1}} > 1 \text{ or } < 1$$

Then the test is known as

- (i) comparison test
- (ii) Raabe's test
- (iii) Cauchy's condensation test
- (iv) D'Alembert's test

(d) The series

$$\Sigma u_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

is

- (i) absolute convergent
- (ii) conditional convergent
- (iii) Both (i) and (ii)
- (iv) None of the above

(e) The function $f(x) = |x| - 1$; $x \in \mathbb{R}$ is

- (i) differentiable at $x = 0$
- (ii) not differentiable at $x = 0$
- (iii) continuous at $x = 1$
- (iv) None of the above

(f) To which of the following, Rolle's theorem can be applied?

- (i) $f(x) = \tan x$ in $[0, \pi]$
- (ii) $f(x) = \cos\left(\frac{1}{x}\right)$ in $[-1, 1]$
- (iii) $f(x) = x^2$ in $[2, 3]$
- (iv) $f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$

- (g) The n th divided difference of a polynomial of n th degree is
- (i) always zero
 - (ii) always equal to n
 - (iii) always constant
 - (iv) not defined
- (h) Lagrange's formula is useful for
- (i) interpolation
 - (ii) extrapolation
 - (iii) inverse interpolation
 - (iv) All of the above

2. Answer the following questions in brief : $2 \times 8 = 16$

- (a) Define derived set with examples.
- (b) Define limit superior of a bounded sequence.
- (c) Define Raabe's test.
- (d) Write the statement of L' Hospital rule.
- (e) Write the properties of a continuous function.
- (f) State the Taylor's theorem with the remainder in Cauchy's form.
- (g) Under what situation is the Newton's method of backward difference of interpolation applicable?
- (h) Write the statement of Weddle's rule.

(Continued)

3. Answer any two of the following questions :

- (a) Define a bounded set and bounded sequence. If $\{a_n\}$ is a bounded sequence such that $a_n > 0$ for all $n \in \mathbb{N}$, then show that

$$\lim \left\{ \frac{1}{a_n} \right\} = \frac{1}{\lim a_n}, \text{ if } \lim \overline{a_n} > 0 \quad 2+5=7$$

- (b) State the axioms of an ordered field. Show that the set of real numbers forms a complete ordered field. $2+5=7$

- (c) Define sequence and range of a sequence. Show that

$$\lim_{n \rightarrow \infty} \left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+n}} \right] = 1$$

$$1+1+5=7$$

4. Answer any two of the following questions :

- (a) Define infinite series with examples. Under what condition a geometric series is convergent? Show that the series

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots$$

is not convergent.

$$2+1+4=7$$

- (b) Show that every absolutely convergent series is convergent. Test the convergence of the following series using Raabe's test : 3+4=7

$$\frac{\alpha}{\beta} + \frac{1+\alpha}{1+\beta} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \dots$$

- (c) State the Leibnitz's test for the convergence of alternating series. If the limit of

$$\frac{\sin 2x - a \sin x}{x^3}$$

as $x \rightarrow 0$ is finite, then find the value of a and the limit. 2+5=7

5. Answer any *two* of the following questions :

- (a) Define uniform continuity of a function. Find the values of a and b , so that $f(x)$ may be differentiable at $x=1$, where

$$f(x) = \begin{cases} x^2 + 3x + a & ; \text{ if } x \leq 1 \\ bx + 2 & ; \text{ if } x > 1 \end{cases} \quad 2+5=7$$

- (b) Give the statement of Rolle's theorem. Show that

$$\frac{v-\mu}{1+v^2} < \tan^{-1} v - \tan^{-1} \mu < \frac{v-\mu}{1+\mu^2}$$

if $0 < \mu < v$ and deduce that

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6} \quad 2+5=7$$

- (c) State Cauchy's mean value theorem. Expand $\cos x$ in powers of x in infinite series using Maclaurin's series expansion. 2+5=7

6. Answer any two of the following questions :

- (a) Define the operators Δ and E . Show that if $h = 1$

$$\Delta^2 \log x = \log \left[1 - \frac{1}{(x+1)^2} \right]$$

Represent the function $f(x)$ given by

$$f(x) = 2x^4 - 12x^3 + 24x^2 - 30x + 9$$

and its successive differences in factorial notation. 1+3+3=7

- (b) What do you mean by interpolation? When would you recommend the formula involving divided differences? Find the third divided difference with arguments 2, 4, 9, 10 of the function

$$f(x) = x^3 - 2x \quad \text{2+2+3=7}$$

- (c) What do you mean by numerical integration? Solve

$$u_{x+1} - au_x = 0; a \neq 1$$

Evaluate $\sqrt{12}$ by applying Newton's formula. 2+2+3=7
