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6 SEM TDC DSE MTH (CBCS) 3 (H)

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(May)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–3

(Discrete Mathematics)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) Define order on a set. Give an example of an ordered set. 1+1=2

(b) Let P and Q be finite ordered sets and let $\psi : P \to Q$ be a bijective map. Then show that the following are equivalent : 3

(i) ψ is an order-isomorphism

(ii) $x < y$ in P iff $\psi(x) < \psi(y)$ in Q

(iii) $x \prec y$ in P iff $\psi(x) \prec \psi(y)$ in Q

(c) In which of the following cases is the map $\phi : P \to Q$ order-preserving? 5

(i) $P = Q = \langle \mathbb{Z}; \leq \rangle$, and $\phi(x) = x + 1$

(ii) $P = \langle \wp(S); \subseteq \rangle$ with $|S| > 1$, $Q = 2$ and $\phi(U) = 1$ if $U \neq \{\}$ and $\phi(\{\}) = 0$

Or

Prove that, for all ordered sets P, Q and R

$$\langle P \to \langle Q \to R \rangle \rangle \cong \langle P \times Q \to R \rangle$$

2. (a) Let P be a lattice. Then for all $a, b, c, d \in P$, show that—

(i) $a \leq b \Rightarrow a \vee c \leq b \vee c$ and $a \wedge c \leq b \wedge c$

(ii) $a \leq b$ and $c \leq d \Rightarrow a \vee c \leq b \vee d$ 2

(b) Let L and K be lattices and $f : L \to K$ a map. Then show that the following are equivalent : 5

(i) f is order-preserving

(ii) $(\forall\, a, b \in L)\ f(a \vee b) \geq f(a) \vee f(b)$

(iii) $(\forall\, a, b \in L)\ f(a \wedge b) \leq f(a) \wedge f(b)$

(c) Let P be a non-empty ordered set. Then prove that P is a complete lattice iff $\wedge S$ exists in P for every subset S of P. 3

(d) Show that the ordered subset

$Q = \{1, 2, 4, 5, 6, 12, 20, 30, 60\}$ of $\langle \mathbb{N}_0; \leq \rangle$

is not a lattice. Draw a diagram of Q and find the elements $a, b, c, d \in Q$ such that $a \vee b$ and $c \wedge d$ do not exist in Q. 5

Or

Give an example of an ordered set P in which there are three elements x, y, z such that—

(i) $\{x, y, z\}$ is an antichain

(ii) $x \vee y$, $y \vee z$ and $z \vee x$ fail to exist

(iii) $\vee\{x, y, z\}$ exists

3. (a) Let L be a lattice. Prove that L is a chain iff every non-empty subset of L is a sublattice. 2

(b) Let $f : L \to K$ be a lattice homomorphism.

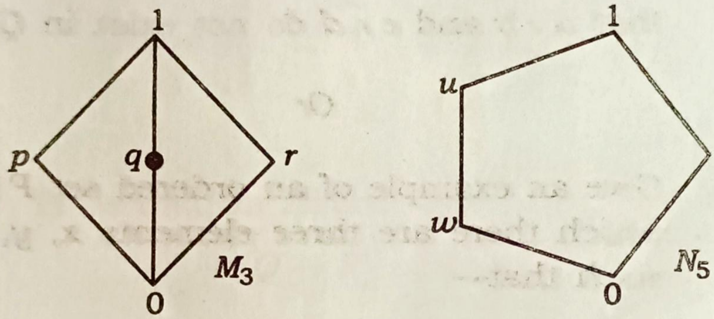
(i) Show that if $M \in \text{sub}\, L$, then $f(M) \in \text{sub}\, K$.

(ii) Show that if $N \in \text{sub}\, K$, then $f^{-1}(N) \in \text{sub}_0\, L$. 2+2=4

(c) Let L be a lattice. Prove that—

(i) L is non-modular iff $N_5 \rightarrowtail L$

(ii) L is non-distributive iff $N_5 \rightarrowtail L$ or $M_3 \rightarrowtail L$, where



3+3=6

(d) Determine the lattices L and K to within isomorphism given that—

(i) L is non-distributive

(ii) K has at least 3 elements

(iii) $|L \times K| = 18$ 3

4. Answer any *two* of the following questions : 5×2=10

(a) Let L be a distributive lattice and let $a, b, c \in L$. Prove that

$$(a \vee b = c \vee b \text{ and } a \wedge b = c \wedge b) \Rightarrow a = c$$

(b) Show that the following hold in all Boolean algebras :

(i) $(a \wedge b) \vee (a' \wedge b) \vee (a \wedge b') \vee (a' \wedge b') = 1$

(ii) $a = b \Leftrightarrow (a \wedge b') \vee (a' \wedge b) = 0$

(c) Let $f : B \to C$, where B and C are Boolean algebras.

(i) Assume that f is a lattice homomorphism. Then show that the following are equivalent :

(1) $f(o) = 0$ and $f(1) = 1$

(2) $f(a') = (f(a))' \ \forall \ a \in B$

(ii) If f preserves '′', then show that f preserves $\vee$ iff f preserves $\wedge$.

5. *(a)* Define a bipartite graph. Find the maximum number of edges of a complete bipartite graph with 6 vertices.

1+2=3

(b) Answer any *three* of the following questions : 3×3=9

(i) Show that the number of odd degree vertices in a graph is always even.

(ii) Find a connected graph whose adjacency matrix is singular.

(iii) Define isomorphism of graphs. Give an example.

(iv) If G is a simple graph with at least two vertices, then prove that G must contain two or more vertices of the same degree.

6. Answer any *three* of the following questions : $6\times3=18$

(a) Prove that a connected graph G is Eulerian iff the degree of each vertex of G is even.

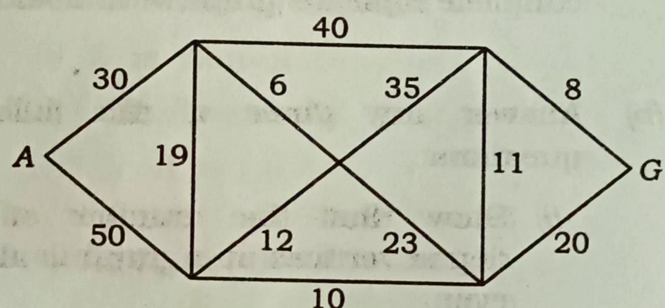
(b) Which of the following graphs are Hamiltonian?

(i) The complete graph, K_5

(ii) The complete bipartite graph, $K_{2,3}$

(iii) The wheel W_6

(c) Find a shortest path from A to G in the following weighted graph :



(d) Write a short note on travelling salesman problem.

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