Total No. of Printed Pages—7

2 SEM TDC MTMH (CBCS) C 4

2024

(May)

MATHEMATICS

(Core)

Paper : C–4

(Differential Equations)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Define particular solution of differential equation. 1

(b) Write the degree of the differential equation

$$\frac{d^4y}{dx^4} - \sin\left(\frac{d^3y}{dx^3}\right) = 0$$ 1

(c) Show that $f(x) = 2e^{3x} - 5e^{4x}$ is a solution of the differential equation

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$$ 3

(d) Solve the initial value problem : 2

$$\frac{dy}{dx} = -\frac{x}{y}, \; y(3) = 4$$

(e) Solve : 2

$$(x + y + 1)\frac{dy}{dx} = 1$$

(f) Solve any *two* from the following : 3×2=6

(i) $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

(ii) $x\frac{dy}{dx} + (x + 1)y = x^3$

(iii) $(x^2 + y^2)dx - 2xy\, dy = 0, \; y(1) = 2$

2. (a) Draw the input-output compartmental diagram for exponential growth model. Write the word equation for density dependent population growth model. 1+1=2

(b) Write the differential equation for the case of a single fast dissolving pill. 2

(c) Answer any *two* from the following : 3×2=6

(i) In a sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. Find what % of the original radioactive nuclei will remain after 200 years.

(ii) The differential equation

$$\frac{dC}{dt} = \frac{F}{V}(C_{in} - C)$$

describes the level of pollution in the lake, where V is the volume of the lake, F is the flow (in and out), C is the concentration of pollution at time t and C_{in} is the concentration of pollution entering the lake. If $V = 28 \times 10^6 \text{ m}^3$, $F = 4 \times 10^6 \text{ m}^3/\text{month}$, find how long would it take for the lake with pollution concentration 10^7 parts/m^3 to drop below the safety threshold ($4 \times 10^6 \text{ parts/m}^3$) if only freshwater enters the lake.

(iii) Derive the differential equation of exponentially growth population model.

3. (a) Define linear homogeneous differential equation. 1

(b) Check whether the functions x and $x \sin x$ are linearly independent or not. 2

(c) Show that e^x and e^{3x} are the solutions of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$$

Also write the linear combinations of the above solution. 3

(d) Answer any *one* of the following : 4

(i) If $y = \frac{1}{x}$ is a solution of

$$x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = 0$$

then find the general solution.

(ii) Solve :

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2e^{3x}$$

4. Answer any *three* from the following : 5×3=15

(a) Solve :

$$\frac{d^2y}{dx^2} + ay = \sec ax$$

(b) Solve :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \sin \log x^2$$

(c) Solve by method of undetermined coefficient :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^{4x}$$

(d) Solve by method of variation of parameter :

$$\frac{d^2y}{dx^2} + y = \tan x$$

5. (a) Define equilibrium solution of a differential equation. 1

(b) Write the word equation and differential equation for predator-prey model. 2

(Turn Over)

6. Answer any *one* of the following : 7

(a) A model for the spread of a disease, where once susceptible infected, confers life-long immunity, is given by the differential equations

$$\frac{dS}{dt} = -\beta SI, \quad \frac{dI}{dt} = \beta SI - \alpha I$$

where α and β are positive constants, $S(t)$ denotes the number of susceptibles and $I(t)$ denotes the number of infectives at time t.

(i) Use the chain rule to find a relation between S and I, given the initial number of susceptibles and infectives are s_0 and i_0, respectively.

(ii) Find and sketch directions of trajectories in the phase plane.

(b) A simple model for a battle between two armies red and blue, where both the army used aimed fire, is given by the coupled differential equations

$$\frac{dR}{dt} = -a_1 B, \quad \frac{dB}{dt} = -a_2 R$$

where R and B are the number of soldiers in the red and blue army respectively and a_1 and a_2 are positive constants.

(i) Use the chain rule to find a relation between R and B, given the initial number of soldiers for the two armies are r_0 and b_0, respectively.

(ii) Draw a rough sketch of phase-plane trajectories.

★ ★ ★