2 SEM TDC STSH (CBCS) C 3 (N/O)

2024

(May)

STATISTICS

(Core)

Paper : C–3

(Probability and Probability Distributions)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the following alternatives : 1×5=5

(a) If A and B are two independent events, then $P(\bar{A} \cap \bar{B})$ is equal to

(i) $P(\bar{A}) \cdot P(\bar{B})$

(ii) $1 - P(A \cup B)$

(iii) $[1 - P(A)] \cdot [1 - P(B)]$

(iv) All of the above

(b) If X and Y are two random variables with means $\overline{X}$ and $\overline{Y}$ respectively, then the expression $E[(X-\overline{X})(Y-\overline{Y})]$ is called

(i) variance of X

(ii) variance of Y

(iii) covariance of X and Y

(iv) moment of X and Y

(c) If X_1 and X_2 are independent random variables, then $M_{X_1+X_2}(t)$ is equal to

(i) $M_{X_1}(t)+M_{X_2}(t)$

(ii) $M_{X_1}(t)-M_{X_2}(t)$

(iii) $M_{X_1}(t)\cdot M_{X_2}(t)$

(iv) None of the above

(d) The mean of the binomial distribution

$$P(X=x)={}^{12}C_x\left(\frac{2}{3}\right)^x\left(\frac{1}{3}\right)^{12-x};\ x=0, 1, 2, \cdots, 12$$

is

(i) 8

(ii) 12

(iii) $\frac{2}{9}$

(iv) $\frac{1}{2}$

(e) The probability that a normal variate X lies within three times of SD from each side of the mean is

(i) 0·9973

(ii) 0·9937

(iii) 0·0027

(iv) 0·9544

2. Answer the following questions : $2 \times 5 = 10$

(a) Write down the classical definition of probability.

(b) A and B are events, such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{2}{3}$. Find $P(B)$.

(c) What are probability mass function and probability density function?

(d) Obtain the m.g.f. of binomial distribution.

(e) Prove that if X and Y are independent random variables, then $E(X+Y) = E(X) + E(Y)$.

3. (a) State and prove Bayes' theorem. 2+4=6

Or

(b) The probabilities of X, Y and Z becoming manager are $\frac{4}{9}$, $\frac{2}{9}$ and $\frac{1}{3}$ respectively. The probabilities that bonus scheme will be introduced if X, Y and Z become manager are $\frac{3}{10}$, $\frac{1}{2}$ and $\frac{4}{5}$ respectively. If bonus scheme has been introduced, what is the probability that the manager appointed is X? 6

4. (a) Let X is a continuous random variable with probability density function

$$\begin{aligned} f(x) &= ax \quad &;\ 0 \le x \le 1 \\ &= a \quad &;\ 1 \le x \le 2 \\ &= -ax + 3a \quad &;\ 2 \le x \le 3 \\ &= 0 \quad &,\ \text{elsewhere} \end{aligned}$$

(i) Determine the value of a.

(ii) Compute $P(X \le 2 \cdot 5)$. 4+4=8

Or

(b) (i) What is meant by conditional probability? What are its properties? 3

(ii) The joint probability distribution of two random variables X and Y is given below. Find the marginal distribution of X and Y. 2½+2½=5

Y \ X	1	2	3
1	0	$\frac{1}{8}$	$\frac{2}{8}$
2	$\frac{1}{8}$	$\frac{1}{16}$	0
3	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{2}{8}$

5. *(a)* Define mathematical expectation of discrete and continuous random variables. A continuous random variable X has a probability density function

$$f(x) = kx^2 e^{-x};\ x > 0$$

find k, $E(X)$ and $V(X)$. 2+2+2+2=8

Or

(b) Define moment generating function and cumulant generating function. Prove that the moment generating function of the sum of a number of independent random variables is equal to the product of their respective moment generating functions. 2+2+4=8

6. *(a)* State and prove the reproductive property of Poisson distribution. Show that the mean and variance of the Poisson distribution are equal. 2+3+4=9

Or

(b) Define geometric distribution and find its m.g.f. Derive the mean and variance of the geometric distribution from its m.g.f. 2+3+4=9

7. *(a)* Under what conditions, does the binomial distribution tend to normal distribution? What is the m.g.f. of normal distribution? State some important properties of normal distribution. 2+2+5=9

Or

(b) Define gamma distribution. State and prove the additive property of gamma distribution. 2+2+5=9

(Old Course)

Full Marks : 50

Pass Marks :20

Time : 2 hours

1. Choose the correct answer from the following alternatives : 1×5=5

   (a) If A and B are two independent events, then $P(\overline{A} \cap \overline{B})$ is equal to

   (i) $P(\overline{A}) \cdot P(\overline{B})$

   (ii) $1 - P(A \cup B)$

   (iii) $[1 - P(A)] \cdot [1 - P(B)]$

   (iv) All of the above

   (b) If X and Y are two random variables with means $\overline{X}$ and $\overline{Y}$ respectively, then the expression $E[(X - \overline{X})(Y - \overline{Y})]$ is called

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   (c) If X_1 and X_2 are independent random variables, then $M_{X_1+X_2}(t)$ is equal to

   (i) $M_{X_1}(t) + M_{X_2}(t)$

   (ii) $M_{X_1}(t) - M_{X_2}(t)$

   (iii) $M_{X_1}(t) \cdot M_{X_2}(t)$

   (iv) None of the above

(d) The mean of the binomial distribution

$$P(X=x)={}^{12}C_x\left(\frac{2}{3}\right)^x\left(\frac{1}{3}\right)^{12-x};\ x=0,1,2,\cdots,12$$

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Determine the value of a. 4

Or

(b) The joint probability distribution of two random variables X and Y is given below. Find the marginal distribution of X and Y.

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