Total No. of Printed Pages—15

2 SEM TDC STSH (CBCS) C 4 (N/O)

2024

(May)

STATISTICS

(Core)

Paper : C–4

(Algebra)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the given alternatives in each of the following : $1 \times 6 = 6$

(a) The union of two subspaces of a vector space $V(F)$

(i) is also a subspace of $V(F)$ iff one is contained in the other

(ii) may not be a subspace of $V(F)$

(iii) Both (i) and (ii) are true

(iv) Both (i) and (ii) are false

(b) If

$$\begin{bmatrix} 5 & k+2 \\ k+1 & -2 \end{bmatrix} = \begin{bmatrix} k+3 & 4 \\ 3 & -k \end{bmatrix}$$

then k is equal to

(i) 0

(ii) 2

(iii) –2

(iv) 1

(c) A polynomial of three terms is called a

(i) monomial

(ii) binomial

(iii) trinomial

(iv) All of the above

(d) The value of the determinant

$$\begin{vmatrix} \frac{1}{a} & 1 & bc \\ \frac{1}{b} & 1 & ca \\ \frac{1}{c} & 1 & ab \end{vmatrix}$$

is

(i) abc

(ii) $\frac{1}{abc}$

(iii) 0

(iv) None of the above

(e) Rank (AA') is equal to

(i) rank A

(ii) rank A'

(iii) 1

(iv) None of the above

(f) The matrix of a quadratic form is

(i) symmetric

(ii) asymmetric

(iii) orthogonal

(iv) Hermitian

2. Answer the following questions : 3×5=15

(a) Find the values of A, B and C for which

$$A(x-3)(x-1)+B(x+1)(x-1)+C(x+1)(x-3)=6x-10$$

is an identity.

(b) Write the properties of a symmetric matrix.

(c) Show that a skew-symmetric determinant of odd order vanishes.

(d) Show that

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ca \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

(e) Define row rank and column rank of a matrix respectively.

3. Answer any *one* of the following questions : 6

(a) The roots of the equation

$$x^3-3x^2+kx+3=0$$

are in AP. Find the value of k and solve the equation.

(b) Discuss the Cardon's method for solving a cubic equation.

(c) Prove that if two vectors are linearly dependent, one of them is a scalar multiple of the other.

4. Answer any *two* of the following questions : 3×2=6

(a) If A and B are two idempotent matrices, then show that $A+B$ will be idempotent if $AB=BA=0$.

(b) Show that every diagonal element of a Hermitian matrix must be real.

(c) Let A and B be two square matrices of order n. Then prove that

$$\text{tr}(AB)=\text{tr}(BA)$$

5. Answer any *two* of the following questions :

(a) Define adjoint of a square matrix. If $A_{n\times n}$ is a square matrix of order n, then prove that

$$A(\text{adj } A)=(\text{adj } A)A=|A|I_n$$ 2+4=6

(b) Discuss the working rule for finding the solution of the equation $AX = B$. 6

(c) Given that

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 1 \\ 0 & 5 & -2 \end{bmatrix}$$

compute—

(i) $|A|$;

(ii) adj A;

(iii) A^{-1}. 2×3=6

6. Answer any *two* of the following questions : 5×2=10

(a) Find the rank of

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 6 & 9 \end{bmatrix}$$

(b) Obtain a *g*-inverse of the matrix given by

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

[illegible] Show that the characteristic roots of an idempotent matrix are either zero or unity.

[illegible] State and prove Cayley-Hamilton theorem.

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. Choose the correct answer from the given alternatives in each of the following : 1×8=8

(a) The fundamental theorem of algebra states that every algebraic equation has

(i) at least one root, real or imaginary

(ii) only two roots, both are real

(iii) only two roots, one real and other imaginary

(iv) at least one root, which is real

(b) The union of two subspaces of a vector space $V(F)$

(i) is also a subspace of $V(F)$ iff one is contained in the other

(ii) may not be a subspace of $V(F)$

(iii) Both (i) and (ii) are true

(iv) Both (i) and (ii) are false

(c) If

$$\begin{bmatrix} 5 & k+2 \\ k+1 & -2 \end{bmatrix} = \begin{bmatrix} k+3 & 4 \\ 3 & -k \end{bmatrix}$$

then k is equal to

(i) 0

(ii) 2

(iii) −2

(iv) 1

(d) If each element of a determinant of third order with value A is multiplied by 3, then the value of newly formed determinant is

(i) $3A$

(ii) $9A$

(iii) $27A$

(iv) None of the above

(e) The value of the determinant

$$\begin{vmatrix} \frac{1}{a} & 1 & bc \\ \frac{1}{b} & 1 & ca \\ \frac{1}{c} & 1 & ab \end{vmatrix}$$

is equal to

(i) abc

(ii) $\frac{1}{abc}$

(iii) 0

(iv) None of the above

(f) A system of m homogeneous linear equations $AX = 0$ in n unknowns has only trivial solution if

(i) $m = n$

(ii) $m \neq n$

(iii) rank $A = m$

(iv) rank $A = n$

(g) If I_3 is identity matrix of order 3, then $(I_3)^{-1}$ is equal to

(i) 0

(ii) $3I_3$

(iii) I_3

(iv) Not necessarily exists

(h) The matrix of a quadratic form is

(i) symmetric

(ii) anti-symmetric

(iii) orthogonal

(iv) Hermitian

2. Answer the following questions : 4×6=24

(a) Find the values of A, B and C for which

$$A(x-3)(x-1)+B(x+1)(x-1)+C(x+1)(x-3)=6x-10$$

is an identity.

(b) Show that the vectors (1, 2, 1), (2, 1, 0) and (1, −1, 2) form a basis of R^3.

(c) Write the properties of a symmetric matrix.

(d) Show that a skew-symmetric determinant of odd order vanishes.

(e) Show that

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ca \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

(f) Show that if A and B be two matrices of the same type, then

$$\rho(A+B) \leq \rho(A) + \rho(B)$$

3. Answer any *two* of the following questions : 5×2=10

(a) The roots of the equation

$$x^3 - 3x^2 + kx + 3 = 0$$

are in AP. Find the value of k and solve the equation.

(b) If the sum of two roots of the cubic

$$x^3 + a_1x^2 + a_2x + a_3 = 0$$

is zero, then prove that $a_1a_2 = a_3$.

(c) Discuss the Cardon's method for solving a cubic equation.

(d) Prove that if two vectors are linearly dependent, then one of them is a scalar multiple of the other.

4. Answer any *two* of the following questions : 5×2=10

(a) If

$$\begin{bmatrix} A^{-1} & 0 \\ X & A^{-1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ B & A \end{bmatrix}^{-1}$$

and A is non-singular, then find out the value of X.

(b) If A and B are two n-rowed orthogonal matrices, then AB and BA are also orthogonal matrices.

(c) Prove $\text{tr}(AB) = \text{tr}(BA)$ if A and B are two square matrices of order n.

(d) Show that

$$A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$

is involutary.

5. Answer any *three* of the following questions :

(a) Define adjoint of a square matrix. If $A_{n \times n}$ is a square matrix of order n, then prove that

$$A(\text{adj } A) = (\text{adj } A)A = |A| I_n$$ 2+4=6

(b) Show that

$$\Delta = \begin{vmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & 3 \\ x & y & 3 & 0 \end{vmatrix} = (x - 2y + 3)^2$$

6

(c) Discuss the working rule for finding the solution of the equation $AX = B$. 6

(d) Explain circulant determinant and Vandermonde determinant for *n*th-order. 6

(e) Given that

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 1 \\ 0 & 5 & -2 \end{bmatrix}$$

compute—

(i) $|A|$;

(ii) adj A;

(iii) A^{-1}. 2×3=6

6. Answer any *two* of the following questions : $5\times2=10$

(a) Find the rank of

$$A=\begin{bmatrix}0 & 2 & 3\\0 & 4 & 6\\0 & 6 & 9\end{bmatrix}$$

(b) Obtain a g-inverse of the matrix given by

$$A=\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}$$

(c) Show that the characteristic roots of an idempotent matrix are either zero or unity.

(d) State and prove Cayley-Hamilton theorem.

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