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2 SEM TDC STSH (CBCS) C 4

2022

(June/July)

STATISTICS

(Core)

Paper : C–4

(Algebra)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the given alternatives in each question : 1×8=8

(a) A polynomial $f(x)$ of nth degree $(n \geq 1)$ with its coefficients real or complex has

(i) exactly n roots

(ii) less than n roots

(iii) greater than n roots

(iv) None of the above

(b) The union of two subspaces of a vector space $V(F)$

(i) is also a subspace of $V(F)$, if and only if one is contained in the other

(ii) may not be a subspace of $V(F)$

(iii) Both (i) and (ii) are true

(iv) Both (i) and (ii) are false

(c) Two matrices are said to be equal, if

(i) they are square matrices

(ii) they are of the same size

(iii) they are of different sizes

(iv) None of the above

(d) If A and B are Hermitian or skew-Hermitian matrices, then

(i) $(A+B)$ is a Hermitian matrix

(ii) $(A+B)$ is a skew-Hermitian

(iii) Both (i) and (ii)

(iv) None of the above

(e) A determinant changes its sign when

(i) two rows are interchanged

(ii) two columns are interchanged

(iii) rows are interchanged with columns

(iv) Both (i) and (ii)

(f) A skew-symmetric determinant

(i) of odd order vanishes

(ii) of even order is a perfect square

(iii) Both (i) and (ii)

(iv) None of the above

(g) For a square matrix A

(i) $A(\text{adj}A) = \frac{(\text{adj}A)}{A}$

(ii) $A(\text{adj}A) = \frac{|A|}{(\text{adj}A)}$

(iii) $A(\text{adj}A) = (\text{adj}A)A = |A|I_n$

(iv) None of the above

(h) The rank of the transpose of a matrix is

(i) same as that of the original matrix

(ii) twice the rank of the original matrix

(iii) proportional to the rank of the original matrix

(iv) None of the above

2. Answer the following : 4×4=16

(a) Solve the equation

$$x^4 - 7x^3 + 27x^2 - 47x + 26 = 0,$$

given that one of its roots is $2+3i$.

(b) Define trace of a square matrix. State its properties.

(c) Prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)$$

(d) Give the definition of rank of a matrix. What do you mean by nullity of a matrix?

3. Answer any *two* of the following : 7×2=14

(a) (i) State the fundamental theorem of classical algebra. 3

(ii) Find the values of A, B, C for which

$$A(x-3)(x-1)+B(x+1)(x-1)$$
$$+C(x+1)(x-3)=6x-10$$

is an identity. 4

(b) Define vector subspace. Let R be the field of real numbers. Which of the following are subspaces of $V_3(R)$? 7

(i) $\{(x, 2y, 3z) : x, y, z \in R\}$

(ii) $\{(x, x, x) : x \in R\}$

(iii) $\{(x, y, z) : x, y, z$ are rational numbers$\}$

(c) In $V_3(R)$, where R is the field of real numbers, examine each of the following sets of vectors for linear dependence : 7

(i) $\{(-1, 2, 1), (3, 0, -1), (-5, 4, 3)\}$

(ii) $\{(2, 3, 5), (4, 9, 25)\}$

(d) Determine whether or not the following vectors form a basis of R^3 : 7

$$(1, 1, 2), (1, 2, 5), (5, 3, 4)$$

4. Answer any *one* of the following : 9

(a) Define an orthogonal matrix. Write the properties of an orthogonal matrix. Show that if A is an orthogonal matrix, then $|A| = \pm 1$. 3+3+3=9

(b) Define singular and non-singular matrices. For what value of x, the following matrix becomes singular? 2+2+5=9

$$\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$$

(c) Define nilpotent matrix. Write the properties of nilpotent matrix. Show that

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

is nilpotent. 3+3+3=9

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5. Answer any *one* of the following :

(a) (i) Prove that if a determinant has two identical rows (columns) its value is zero. 3

(ii) Show that

$$\Delta \equiv \begin{vmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix} = (x-2y+z)^2$$

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(iii) What do you mean by conjugate element of a determinant? Define symmetric determinant. 2+2=4

(b) (i) Define inverse of a matrix. What is the necessary and sufficient condition for a square matrix A to possess the inverse? 2+2=4

(ii) Find the adjoint of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

and verify the theorem

$$A(\text{adj}A) = (\text{adj}A)A = |A| I_n$$

4+3=7

6. Answer any *two* of the following : 6×2=12

(a) Investigate for what values of λ and μ, the system of simultaneous equations

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

has (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions. 6

(b) What is echelon matrix? Reduce the following matrix into echelon form : 2+4=6

$$A=\begin{bmatrix}1 & 2 & -3 & 1 & 2\\ 2 & 4 & -4 & 6 & 10\\ 3 & 6 & -6 & 9 & 13\end{bmatrix}$$

(c) Show that

$$\Delta_{n+1}=\begin{vmatrix}x & a & a & \cdots & a\\ a & x & a & \cdots & a\\ \vdots & \vdots & \vdots & & \vdots\\ a & a & a & \cdots & x\end{vmatrix}=(x+na)(x-a)^n$$

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7. Answer any *two* of the following : 5×2=10

(a) Show that the characteristic roots of an idempotent matrix are either zero or unity. 5

(b) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

and hence compute A^{-1}. 5

(c) Write the quadratic form corresponding to the symmetric matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix}$$

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(d) What do you mean by g-inverse? Show that a generalised inverse always exists and is not unique. 5

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