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1 SEM TDC STSH (CBCS) C 2 (N/O)

2022

(Nov/Dec)

STATISTICS

(Core)

Paper : C–2

(Calculus)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

(New Course)

1. Choose the correct answer from the following alternatives in each question : 1×8=8

(a) The value of $\lim\limits_{x \to 0} \dfrac{x}{\sin x}$ is

(i) 0

(ii) 1

(iii) ∞

(iv) None of the above

(b) Let $P(x, y)$ be a point on the curve $y = f(x)$. Then the curve at P is concave w.r.t. to X-axis, if

(i) $y\frac{d^2y}{dx^2}$ is positive at P

(ii) $y\frac{d^2y}{dx^2}$ is negative at P

(iii) $y\frac{dy}{dx}$ is positive at P

(iv) $y\frac{dy}{dx}$ is negative at P

(c) A function $f(x, y)$ is said to be homogeneous of degree n in the variables x and y, if it can be expressed in the form

(i) $x^n \phi\left(\frac{y}{x}\right)$

(ii) $y^n \phi\left(\frac{x}{y}\right)$

(iii) Both (i) and (ii)

(iv) None of the above

(d) The value of the integral $\int \frac{f'(x)}{f(x)} dx$ is

(i) $\log f(x)$

(ii) $\log f'(x)$

(iii) $\log |f(x)|$

(iv) None of the above

(e) The value of $\int_0^4 \int_0^1 xy(x-y)\, dy\, dx$ is

(i) 2

(ii) 4

(iii) 6

(iv) 8

(f) The differential equation $M\,dx + N\,dy = 0$ is exact, if and only if

(i) $\frac{\partial M}{\partial x} = \frac{\partial M}{\partial y}$

(ii) $\frac{\partial^2 M}{\partial y^2} = \frac{\partial^2 N}{\partial x^2}$

(iii) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(iv) $\frac{\partial^2 M}{\partial x^2} = \frac{\partial^2 N}{\partial y^2}$

(g) A solution of a differential equation which contains no arbitrary constants is

(i) particular solution

(ii) general solution

(iii) primitive solution

(iv) None of the above

(h) Elimination of a function f from $z = f\left(\frac{y}{x}\right)$ gives a partial differential equation

(i) $x\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$

(ii) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$

(iii) $\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$

(iv) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$

2. Answer the following questions :

(a) Evaluate : 3

$$\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$$

(b) If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r\cos\theta$, then show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta$$ 3

(c) Prove that

$$\beta(m, n) = \beta(m+1, n) + \beta(m, n+1)$$ 4

(d) Solve

$$(x+y)^2 \frac{dy}{dx} = k^2$$

where k is a constant. 4

(e) Form the partial differential equation by eliminating arbitrary constants a and b from

$$z = ax + by + ab$$ 2

3. Answer any *three* of the following questions : 7×3=21

(a) Define continuity of a function at the end points. Examine the continuity of the function

$$f(x) = \begin{cases} -x^2 & , \quad x \le 0 \\ 5x-4 & , \quad 0 < x \le 1 \\ 4x^2 - 3x & , \quad 1 < x < 2 \\ 3x+4 & , \quad x \ge 2 \end{cases}$$

at $x = 0, 1, 2$.

(b) If $u = r^3$, $x^2 + y^2 + z^2 = r^2$, then prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 12r$$

(c) If $y = e^{ax} \sin bx$, then find y_n.

(d) State and prove Leibnitz's rule for successive differentiation.

(e) State Euler's theorem on homogeneous function. If

$$u = \tan^{-1} \frac{x^3 + y^3}{x - y}$$

then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

(f) For what value of x, the following expression is maximum and minimum respectively?

$$2x^3 - 21x^2 + 36x - 20$$

Also, find the maximum and minimum values of the expression.

4. Answer any *two* of the following questions : 7×2=14

(a) Show that

$$\int_0^{\pi/4} \log(1 + \tan\theta)\, d\theta = \frac{\pi}{8} \log 2$$

(b) Find :

$$\int_0^1 \int_0^x \frac{y\, dx\, dy}{\sqrt{x^2 + y^2}}$$

(c) Define differentiating under the integral sign. Evaluate the following integral by introducing a parameter and carrying out a suitable differentiation under the integral sign :

$$\int_0^1 x^{4/3} \log_e x \, dx$$

(d) Define gamma and beta functions. Derive the following relation between beta and gamma functions :

$$\beta(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$$

(e) Show that

$$\Gamma(n)\,\Gamma(1-n) = \frac{\pi}{\sin n\pi}; \quad 0 < n < 1$$

Then find the value of $\Gamma(\frac{1}{2})$.

5. Answer any *two* of the following questions :

7×2=14

(a) What is exact differential equation? State the necessary and sufficient condition for the ordinary differential equation to be exact. Show that the differential equation

$$(y^3 + 3x^2 y)\,dx + (x^3 + 3xy^2)\,dy = 0$$

is exact and also solve it.

(b) Write down two rules for finding the integrating factors of differential equation $M\,dx + N\,dy = 0$ to make it exact and solve the following differential equation :

$$(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$$

(c) What is homogeneous linear differential equation? Discuss the working rule for solving a homogeneous linear differential equation and then solve

$$dx - \frac{2xy}{(x^2 + y^2)}\,dy = 0$$

(d) What is total differential equation? Write the integrability condition of total differential equation. Solve

$$dx + dy + (x + y + z)\,dz = 0$$

(e) What is the general form of linear differential equation of order n with constant coefficients? Solve the following differential equation :

$$(D^3 + 3D^2 + 2D)\,y = x^2$$

6. Answer any *one* of the following questions : 7

(a) Define partial differential equation with an example. Solve the following differential equation :

$$\frac{\partial^2 z}{\partial x\,\partial y} = xy^2$$

(b) What is the working rule for solving $P_p + Q_q = R$ by Lagrange's method, where P, Q and R are functions of x, y and z? Solve the following differential equation by Lagrange's method :

$$y^2 p - xyq = x(z - 2y)$$

(c) Find a complete integral of

$$z = px + qy + p^2 + q^2$$

Solve the differential equation

$$pq = k$$

where k is a constant.

(Old Course)

1. Choose the correct answer from the following alternatives in each question : 1×8=8

(a) A function f is said to be continuous at a point c, $a < c < b$, if

(i) $\lim_{x \to c} f(x) = f(c)$

(ii) $\lim_{x \to c} f(x) \neq f(c)$

(iii) $\lim_{x \to c} f(x) = \infty$

(iv) None of the above

(b) If $u = x^4 + x^2y^2 + y^4$, then the value of $\frac{\partial^2 u}{\partial x \, \partial y}$ is

(i) $4x^3 + 2xy$

(ii) $4xy$

(iii) $4x^3y$

(iv) None of the above

(c) If two tangents are coincident, then the double point is called

(i) node

(ii) cusp

(iii) conjugate point

(iv) None of the above

(d) The function $f(x)$ is called even, if

(i) $f(-x) = -f(x)$

(ii) $f(-x) = f(x)$

(iii) $f(-x) = x$

(iv) None of the above

(e) The value of $\Gamma(n+1)$ is

(i) $n!$

(ii) $(n-1)\,\Gamma(n-1)$

(iii) $n\Gamma n$

(iv) None of the above

(f) The differential equation $M\,dx + N\,dy = 0$ is exact, if

(i) $\frac{\partial N}{\partial y} = \frac{\partial M}{\partial x}$

(ii) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(iii) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

(iv) None of the above

(g) The integrating factor of the equation $\frac{dy}{dx}+\frac{y}{x}=y^2$ is

(i) $\log\left(\frac{1}{x}\right)$

(ii) $\frac{1}{x}$

(iii) $\log x$

(iv) None of the above

(h) If $z=ax+by+ab$, then the partial differential equation is

(i) $z=x\frac{\partial z}{\partial y}+y\frac{\partial z}{\partial x}$

(ii) $z=x\frac{\partial z}{\partial y}+\left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)+y\left(\frac{\partial z}{\partial x}\right)$

(iii) $z=x\frac{\partial z}{\partial x}+y\left(\frac{\partial z}{\partial y}\right)+\left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)$

(iv) None of the above

2. Answer the following questions in brief : $2\times8=16$

(a) Write the properties of a continuous function.

(b) Define homogeneous function with examples.

(c) Define convexity and point of inflexion of a function.

(d) Write any two fundamental properties of definite integral.

(e) Express $\int_0^1 \frac{x^2}{\sqrt{1-x^5}} dx$ in terms of beta function.

(f) Discuss the procedure of solving a Bernoulli's equation.

(g) How to reduce a non-homogeneous differential equation to a homogeneous form?

(h) Distinguish between partial differential equation and ordinary differential equation.

3. Answer any *three* of the following questions : 7×3=21

(a) Show that the function defined by

$$f(x) = \begin{cases} (1+2x)^{1/x}, & \text{for } x \neq 0 \\ e^2, & \text{for } x = 0 \end{cases}$$

is continuous at $x = 0$. Evaluate $\lim_{x \to 0} x \log x$. 3+4=7

(b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$$

Verify Euler's theorem for

$$f(x, y) = x^3 + 2x^3 y + y^3$$ 4+3=7

(c) Find for what value(s) of x the function $f(x) = 41 - 72x - 18x^2$ attains its maximum. If $u = \frac{x}{x+y}$, $v = x + y$, then find $J\left(\frac{x, y}{u, v}\right)$. 3+4=7

(d) Using Lagrange's method of undetermined multiplier, find x and y in such a way that $x + y = 100$ and the product xy becomes maximum. 7

4. Answer any *two* of the following questions : 7×2=14

(a) Show that

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{4}$$

Evaluate

$$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx\, dy}{1 + x^2 + y^2}$$ 3+4=7

(b) Change the order of integration and evaluate the integral

$$\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dx\, dy$$ 7

(c) Show that

$$\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{1}{2} B\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$$

$$p > -1,\ q > -1$$

and hence deduce that

$$\int_0^2 x^4 (8 - x^3)^{-1/3} dx = \frac{16}{3} B\left(\frac{5}{3}, \frac{2}{3}\right)$$ 5+2=7

5. Answer any *two* of the following questions : 7×2=14

(a) What is an exact differential equation? Solve

$$(1 + xy)\, y\, dx + (1 - xy)\, x\, dx = 0$$ 1+6=7

(b) Write the general form of a first-order and first-degree differential equation. Discuss the working rule for solving a differential equation of the form

$$f_1(x)\, dx = f_2(y)\, dy$$ 1+6=7

(c) What is a linear differential equation? Write the general form of Clairaut's equation. Solve

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$$ 1+1+5=7

6. Answer any *one* of the following questions : 7

(a) What is Lagrange's equation of linear partial differential equation of order one? Solve the following differential equation by direct integration method : 2+5=7

$$xyz - qy = x^2$$

(b) What is a non-linear partial differential equation? Solve the following differential equation by Charpit's method : 2+5=7

$$xp + yq = pq$$

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