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5 SEM TDC DSE MTH (CBCS)
1.1/1.2/1.3 (H)

2024

(November)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE-1

Full Marks : 80

Pass Marks : 32

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

Paper : DSE-1.1

(ANALYTICAL GEOMETRY)

1. Answer the following questions :

- (a) Write the vertex of the conic
 $(y - 2)^2 = 2(x + 2)$. 1
- (b) Write the processes to sketch the
ellipse. 4

(c) Find the focus, vertex, equation of directrix and length of the latus rectum of the conic $x = y^2 - 4y + 2$. 4

(d) Describe the graph of the curve $3(x-2)^2 + 4(y+1)^2 = 12$. Also find its centre and foci. 6

Or

Describe the graph of the hyperbola $x^2 - y^2 - 4x + 8y - 21 = 0$ and sketch its graph.

2. Answer the following questions :

(a) Write the equation of the tangent to the parabola $x^2 = 4ay$ at the point (x_1, y_1) . 1

(b) Write True or False : 1

An ellipse is the set of all points in the plane that are equidistant from a fixed line and a fixed point not on the line.

(c) Suppose that an ellipse has semi-major axis a , semi-minor axis b and foci $(\pm c, 0)$. Then write the expression c in terms of a and b . 1

(d) Find the equation of the parabola that has its vertex at $(1, 2)$ and focus at $(4, 2)$. Also state the reflection property of the parabola. 6

- (e) Find the equation of the ellipse with foci $(0, \pm 2)$ and major axis with end-points $(0, \pm 4)$ and also sketch it. 6

Or

Find the equation and sketch the curve of the hyperbola whose foci $(-3, -3)$ and $(3, 3)$.

3. Answer the following questions :

- (a) Write the condition that the quadratic equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

represents ellipse. 1

- (b) Find the new coordinates of the point $(2, 4)$ if the coordinate axes are rotated through an angle $\theta = \pi/6$. 2

- (c) Consider the equation $x^2 - xy + y^2 - 6 = 0$. Rotate the coordinate axes to remove the xy -term. Then identify the type of conic represented by the equation and sketch its graph. 6

- (d) Let an $x'y'$ -coordinate system be obtained by rotating an xy -coordinate system through an angle $\theta = 30^\circ$.

- (i) Find the $x'y'$ -coordinate of the point whose xy -coordinates are $(2, 4)$.

(ii) Find an equation of the curve

$$2x^2 + 2\sqrt{3}xy = 3$$

in $x'y'$ -coordinates.

6

Or

Identify and sketch the curve

$$14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$$

4. Answer the following questions :

(a) Write the equation of the sphere whose end-points of the diameter is given. 1

(b) Write the standard equation of hyperbola of one sheet. 1

(c) Write True or False :
Curve of intersection of two spheres is a sphere. 1

(d) Write the centre and radius of the sphere

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0 \quad 2$$

(e) Find the equation of the sphere whose centre is $(2, -3, 4)$ and radius is 6 units. 5

(f) Find the equation of the sphere which passes through the points $(1, -3, 4)$, $(1, -5, 2)$, $(1, -3, 0)$ and whose centre lies on the line $x + y + z = 0$. 5

Or

Find the equation of the sphere of centre at (1, 2, 3) and touching a plane at (2, 1, 3).

5. Answer the following questions :

(a) Find the radius and centre of the circle

$$x^2 + y^2 + z^2 - x - y - z - 1 = 0, \quad x + y + z = 0 \quad 5$$

(b) Find the equation of the sphere for which the circle

$$x^2 + y^2 + z^2 + 10y - 4z - 8 = 0, \quad x + y + z = 3$$

is a great circle. 5

Or

Find the equations of the spheres which pass through the

$$x^2 + y^2 + z^2 - 2x + 2y + 4z - 3 = 0; \quad 2x + y + z - 4 = 0$$

and touch the plane $3x + 4y - 14 = 0$.

6. Answer the following questions :

(a) Show that the plane $2x + y - z = 12$ touches the sphere $x^2 + y^2 + z^2 = 24$ and find its point of contact. 5

(b) Classify and sketch the surface $9x^2 + 4y^2 + z^2 = 36$. 5

Or

Classify and sketch the surface $z = x^2 + 4y^2$.

(6)

Paper : DSE-1.2

(PORTFOLIO OPTIMIZATION)

1. Answer any six of the following questions :
1×6=6

- (a) What is mutual fund?
- (b) Define diversification.
- (c) Write one advantage of Sharpe performance model.
- (d) What is market timing?
- (e) What is beta of a portfolio?
- (f) Define risk-free asset.
- (g) What is the value of correlation between risky asset and risk-free asset?

2. Answer any six of the following questions :
4×6=24

- (a) Write a short note on investment objective and investment constraints. 4
- (b) What are the different components of systematic risk? 4
- (c) How would you differentiate risk from uncertainty? 4
- (d) What is the difference between historical and expected returns? Write two measures of mean historical returns. 3+1=4

- (e) If an investment that costs ₹ 400 and is worth ₹ 500 after being held for two years, find annual holding period return (annual HPR) and annual holding period yield (annual HPY). $2+2=4$
- (f) Discuss four different types of risk of an investment. 4
- (g) Calculate the expected rate of return and the risk in terms of variance of the following economic scenarios : $2+2=4$

<i>Economic Conditions</i>	<i>Probability</i>	<i>Rate of Return</i>
Strong economy	0.25	0.20
Weak economy	0.25	-0.20
No major change in economy	0.50	0.10

3. (a) How can risk of an asset be calculated? 2
- (b) State one-fund theorem. 2
- (c) What do you mean by efficient frontier? 2
4. Answer any *two* of the following questions : $6 \times 2 = 12$
- (a) Write the assumptions of capital market theory. Explain briefly.
- (b) Discuss some of the disadvantages of Markowitz model.
- (c) In what way two-factor model is better than one-factor model? Justify.

5. Describe variance and standard variation of returns for a portfolio of investments.

Or

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Find the covariance of rates of returns of US stocks and US bonds as given below :

2020	US Stock Index (R_i)	US Bond Index (R_j)
January	-3.60	1.58
February	3.10	0.40
March	6.03	-0.85
April	1.58	1.05
May	-7.99	1.71
June	-5.24	1.87
July	7.01	0.68
August	-4.51	2.01
September	8.92	0.02
October	3.81	-0.16
November	0.01	0.70
December	6.68	-1.80

If standard deviations of both scenarios are $\sigma_i = 5.56$ and $\sigma_j = 1.22$, then find the correlation.

4+3=7

6. Write the formula for beta of a portfolio. Interpret beta of 1.20 and 0.70.

1+2=3

7. Answer any *three* of the following questions : 5×3=15

- (a) Distinguish between capital market line (CML) and security market line (SML).
- (b) State the limitations of Jensen's performance index model.
- (c) Discuss the assumptions of capital asset pricing model (CAPM).
- (d) Discuss the effects of combining securities in portfolio.

8. Describe Treynor portfolio performance measure with example. 7

Or

Consider the following information on three mutual funds *P*, *Q* and *R*, and the market index :

	<i>Mean Return</i>	<i>SD</i>	<i>Beta</i>
<i>P</i>	15%	20%	0.90
<i>Q</i>	17%	24%	1.10
<i>R</i>	19%	27%	1.20
<i>Market Index</i>	16%	20%	1.00

The mean risk-free rate is 10%. Calculate the Treynor's measure and Sharpe measure for the three mutual funds. 4+3=7

Paper : DSE-1.3

(FINANCIAL MATHEMATICS)

1. (a) Write the inverse supply function of the function $30q + 5p = 80$. 1
- (b) Write the revenue, if p is the price and q is the number of quantities sold. 1
- (c) For demand and supply functions, the equilibrium set will always a singleton set. State true or false. 1
- (d) After introduction of excise tax, among the supply and demand functions, write which function remains same. 1
- (e) Write two reasons for introducing excise tax. 2
- (f) Find the solution of the recurrence equation $5y_t = 3y_{t-1} + 6$, given $y_0 = 4$. 4

Or

Show that the present value of an annuity I for N years, given the fixed interest rate r is

$$P = \frac{I}{1+r} + \frac{I}{(1+r)^2} + \frac{I}{(1+r)^3} + \dots + \frac{I}{(1+r)^N}$$

2. Answer any *two* of the following : 4×2=8

- (a) Describe cobweb model.
- (b) Describe general linear case.
- (c) Determine whether cobweb model predicts stable or unstable equilibrium for the market with supply and demand functions $2p - 3q = 12$ and $2p + q = 20$ respectively.

3. (a) Define fixed cost. 1

(b) Write one difference between maximum point and a point of inflection. 1

(c) Find the critical points of the function $f(x) = x^3 - 12x^2 + 21x + 12$. 2

(d) Find the extreme values of the function $f(x) = x^4 - 8x^3 + 16x^2 - 7$ in the interval $[1, 4]$. 4

4. (a) Write when demand is elastic. 1

(b) If $C(q) = 100 + 20q - 2q^2 + 6q^3$ be the cost function, then find the fixed cost. 2

(c) Define break-even point and variable cost. 2+2=4

- (d) Show that at the startup point, marginal cost is equal to average variable cost.

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Or

Find the elasticity of demand for the demand function

$$D = \{(q, p) : q(1 + p^2) = 50\}$$

Also find the values of p when the demand is elastic and inelastic.

5. (a) Let

$$f(x, y) = x^3y^2 + x^2y$$

Find

$$\frac{\partial f}{\partial x}$$

1

- (b) Let p_1 and p_2 are the selling prices of two items X and Y . Write the revenue of producing q_1 units of X and q_2 units of Y .

1

- (c) Find the critical point(s) of the profit function

$$I(x, y) = 5x + 24y - 1.5x^2 - 2y^2 + xy - 5$$

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- (d) Classify the critical points of the function $f(x, y) = y^3 + 3xy - x^3$ and find the extreme values.

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Or

A firm produces two goods A and B, with demand functions $x = 12 - p^A$, $y = 18 - p^B$. Firm's cost function is $c(x, y) = x^2 + y^2 + 2xy$. Find the maximum achievable profit.

6. (a) Define portfolio. 1
- (b) Write when a portfolio is called an arbitrage portfolio. 1
- (c) Answer any two questions from the following : $5 \times 2 = 10$

(i) Describe technology matrix.

(ii) Let

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \text{ and } A^n = \begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix}$$

$n \geq 2$. Find recurrence equations for a_n, b_n, c_n, d_n .

(iii) For input-output model with two industries, the coefficient matrix is

$$A = \begin{bmatrix} 0.3 & 0.1 \\ 0.2 & 0.4 \end{bmatrix}$$

Determine the production schedule x in terms of external demand d .

7. (a) Cash flow may be negative. State true or false. 1
- (b) Write any one form of hedging. 1
- (c) Interest may be called by an another name. Write that name. 1
- (d) Write the meaning of the cash flow stream (-100, 200). 1
- (e) Write the risk aversion principle. 2
8. (a) Write under what type of interest, capital exhibits geometric growth. 1
- (b) If r is the 1-year interest rate, then write 1-year discount factor. 1
- (c) Define effective interest rate. 2
- (d) Find the internal rate of return of the cash flow (-1, 1, 0, 1). 5

Or

Describe Macaulay duration.

- (e) Explain callable bonds. 2

- (f) Find the future value of the cash flow stream $(-20, 10, 10, 10)$ when the periods are years and the interest rate is 10%.

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