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5 SEM TDC MTMH (CBCS) C 11

2024

(November)

MATHEMATICS

(Core)

Paper : C-11

(Multivariate Calculus)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. (a) State the range of

$$f(x, y) = \frac{1}{xy} \quad 1$$

- (b) Fill in the blank : 1

The definition of limit of $f(x, y)$ applies to the boundary points and _____ of the domain of f .

(c) State when

$$\lim_{(x, y) \rightarrow (x_0, y_0)} f(x, y)$$

does not exist.

1

(d) Investigate the existence of the limit

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{x^2 + y}{y}$$

2

(e) Find f_x and f_y , given $f(x, y) = \log_y x$.

2

(f) Use chain rule to find $\frac{dw}{dt}$ where
 $w = x^2 + y^2$ along the path $x = \cos t$ and
 $y = \sin t$ at $t = \pi$.

2

(g) If $w = f(x, y)$ is differentiable and both x
and y are differentiable of t , then show
that w is a differentiable function of
 t and

$$\frac{dw}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

3

Or

If $w = yz + zx + xy$ and $x = r + s$,
 $y = r - s$, $z = rs$, then find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$
at $(r, s) = (2, 1)$.

(h) Determine ∇f at $(1, 2, -2)$ where

$$f(x, y, z) = (x^2 + y^2 + z^2)^{-\frac{1}{2}} + \log xyz \quad 4$$

(i) Find the directional derivative of

$$f(x, y, z) = 3e^x \cos yz$$

at the origin in the direction of

$$2\hat{i} + \hat{j} - 2\hat{k} \quad 4$$

Or

Find the tangent plane and normal to the surface $x + y + z = 1$ at $(0, 1, 0)$.

(j) Find the local extrema or saddle point as applicable of the function

$$f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy \quad 5$$

(k) Use the method of Lagrange's multiplier to maximise $f(x, y) = xy$ subject to the constraint $x + y = 16$. 5

Or

Find the points on the surface $z^2 = xy + 4$ closest to the origin.

2. (a) Find the y -limits of integration for the integral

$$\iint_R f(x, y) dA$$

where R is the region bounded by the line $x + y = 1$ and a circle of radius 1 with its centre at the origin. 1

- (b) Sketch the region of integration on the plain paper for the integral

$$\int_0^3 \int_0^2 f(x, y) dx dy \quad 1$$

- (c) Write any one iterated integral for the double integral

$$\iint_R f(x, y) dA$$

where R is the triangular region with the vertices $(0, 0)$, $(1, 0)$ and $(0, 1)$. 2

- (d) Reverse the order of integration

$$\int_0^1 \int_2^{4-2x} dy dx \quad 2$$

- (e) Find the volume of the region between the cylinder $z = y^2$ and xy -plane bounded by the planes $x = 0$, $x = 1$, $y = -1$, $y = 1$. 4

(5)

- (f) Set up the iterated integral for evaluating the integral

$$\iiint_D f(r, \theta, z) dz r dr d\theta$$

over the region D which is a prism with its base on the xy -plane bounded by the x -axis and the lines $y=x$ and $x=1$ and whose top lies in the plane $z=2-y$.

5

- (g) Write any two different iterated triple integrals for determining the volume of the tetrahedron bounded by the coordinate planes and the plane

$$x + \frac{y}{2} + \frac{z}{3} = 1 \qquad 2\frac{1}{2} + 2\frac{1}{2} = 5$$

3. (a) Define Jacobian of the transformation

$$x = f(u, v); \quad y = g(u, v) \qquad 1$$

- (b) If

$$u = \frac{1}{2}(x+1), \quad v = \frac{1}{3}(y+4); \quad w = 2z+4$$

find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. 3

- (c) Evaluate $\int_C (xy + y + z) ds$ along the curve

$$\vec{r}(t) = 2t\hat{i} + t\hat{j} + (2 - 2t)\hat{k}; \quad 0 \leq t \leq 1 \quad 3$$

- (d) Show that the work done $\int \vec{F} \cdot d\vec{r}$ around every closed loop in an open region D is zero if and only if \vec{F} is conservative in D . 4

- (e) State the fundamental theorem on line integrals. If \vec{F} is a vector field whose components are continuous throughout an open connected region D in space, then there exists a differentiable function $f(x, y, z)$ such that $\vec{F} = \nabla f$, show that

$$\int_A^B \vec{F} \cdot d\vec{r}$$

is path independent where

$$\vec{r} = g(t)\hat{i} + h(t)\hat{j} + k(t)\hat{k}; \quad a \leq t \leq b$$

is a smooth curve joining A and B in D .

$$1+3=4$$

4. (a) Find the curl of the function

$$\vec{F}(x, y) = (x^2 - y)\hat{i} + (xy - y^2)\hat{j} \quad 1$$

- (b) State Green's theorem in circulation-curl or tangential form. 2

(c) Integrate $f(x, y, z) = x + y + z$ over the surface of the cube cut from the first octant by the planes $x = a; y = a; z = a$. 3

(d) Evaluate $\iint_S f d\sigma$ where S is the surface area of the cone $z = \sqrt{x^2 + y^2}; 0 \leq z \leq 1$ and $f(x, y, z) = x^2$. 4

(e) State and prove Stokes' theorem. 5

Or

State and prove divergence theorem.
