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1 SEM TDC GEMT (CBCS) GE 1 (A/B/C)

2024

(November)

MATHEMATICS

(Generic Elective)

Paper : GE–1

The figures in the margin indicate full marks for the questions

Paper : GE–1 (A)

(Differential Calculus)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. (a) Write the type of discontinuity if

$$\lim_{x \to c-0} f(x) \neq \lim_{x \to c+0} f(x)$$ 1

(b) Applying (δ, ε) definition, show that

$$\lim_{x \to 4} (2x-2) = 6$$ 2

(c) If $f(x) = \dfrac{2x^2-8}{x-2}$, $L=8$, $a=2$, $\varepsilon = 0 \cdot 1$, then find a δ satisfying

$$|f(x) - L| < \varepsilon \text{ if } 0 < |x-a| < \delta$$ 3

(d) Show that $f(x) = |x|$ is continuous everywhere. 3

Or

A function f is defined on $\mathbb{R}$ by

$$f(x) = \begin{cases} 4x^2 - 3x & \text{if} \quad 1 < x < 2 \\ 3x + 4 & \text{if} \quad x \geq 2 \end{cases}$$

Examine f for continuity at $x = 2$.

2. (a) Prove that if a function f is differentiable at $x = c$, then f is continuous at c. 3

Or

Discuss the derivability of the function

$$f(x) = \begin{cases} 5x - 4, & 0 < x \leq 1 \\ 4x^2 - 3x, & x > 1 \end{cases}$$

at $x = 1$.

(b) If $y = \sin ax$, find y_n. 1

(c) If $y = \sin^3 x$, find y_n. 2

(d) State and prove Leibnitz's theorem. 4

Or

If $y = a\cos(\log x) + b\sin(\log x)$, then show that

$$x^2 y_{n+2} + (2n+1)\, x y_{n+1} + (n^2 + 1)\, y_n = 0$$

3. (a) If $f(x, y) = e^{x^2 + xy + y^2}$, then find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. 2

(b) If

$$u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$$

then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$$ 4

Or

If $u = e^{xyz}$, then prove that

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$$

(c) Define homogeneous function of two variables. 1

(d) State and prove Euler's theorem on homogeneous functions of two variables. 4

Or

If $u = \sin^{-1}\left\{\frac{(x^2 + y^2)}{(x + y)}\right\}$, then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$$

4. (a) Find the slope of the tangent to the curve $y = \frac{1}{x-1}$ at $x = 3$. 1

(b) Find the equation of the tangent to the curve $x^2 + xy - y^2 = 1$ at the point (2, 3). 3

Or

Find the equation of the normal to the curve $x^2 - xy + y^2 = 7$ at (−1, 2).

(c) The position $P(x, y)$ of a particle moving in xy-plane is given by the parametric equations

$$x = a\cos t,\ y = a\sin t; \quad 0 \le t \le 2\pi$$

Identify the particle's path. 2

(d) Graph the parametric equations

$$x = 3t - 4,\ y = 6t + 2$$ 4

Or

Identify the symmetry of $r = 3(1 + \sin\theta)$ and then draw the graph.

5. (a) Find the curvature and radius of curvature for the curve

$$\vec{r}(t) = 5\cos t\hat{i} + 12\sin t\hat{j} + t\hat{k}$$

at $t = \frac{\pi}{2}$. 5

(b) Draw the graph of the equation

$$y = x^3 - 3x + 3$$

and identify the inflection point, if any. 5

Or

Find the asymptotes of the graph of

$$y = \frac{x^2 + 1}{x}.$$

6. (a) State Rolle's theorem. 1

(b) Verify Rolle's theorem for the function

$$f(x) = x^2 - 3x + 2$$

in the interval [1, 2]. 3

(c) State and prove Lagrange's mean value theorem. 5

(d) Find the value of c in the mean value theorem $f(b) - f(a) = (b - a) f'(c)$ when $a = 1,\ b = 2$. 2

7. (a) Find the Taylor series generated by $f(x) = e^x$ at $x = 0$. 3

(b) Write the remainder after n terms of Taylor's series in Cauchy's form. 1

(c) State and prove Taylor's theorem with Cauchy's form of remainder. 5

Or

Using Maclaurin's theorem, expand $\cos x$ in an infinite series in powers of x.

8. *(a)* Define stationary point. 1

(b) Evaluate extreme value of the function

$$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 5$$ 5

(c) Evaluate (any *two*) : 2×2=4

(i) $\lim_{x \to 0} \frac{x - \sin x}{x^3}$

(ii) $\lim_{x \to 0} \frac{\sin 2x - 2\sin x}{x^3}$

(iii) $\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$

Paper : GE–1 (B)

(Object-Oriented Programming in C++)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

1. Answer the following questions briefly : 2×5=10

(a) What are classes and objects in C++?

(b) Write the syntax of for loop and give example.

(c) What are the C++ access modifiers?

(d) What is abstract class?

(e) What are default arguments?

2. Answer the following questions : 3×5=15

(a) How does the compilation process work in C++?

(b) What is the difference between procedural programming and object-oriented programming?

(c) Explain a general structure of C++ program with an example.

(d) What is pointer variable? What are the applications of pointer variable?

(e) Explain the basic data types in C++ with example.

3. Answer any *five* of the following questions : 4×5=20

(a) What is encapsulation and how is it implemented in C++?

(b) What are the differences between Call by Value and Call by Reference?

(c) Explain the different types of branching statement in C++ with syntax.

(d) Differentiate between operator overloading and operator overriding.

(e) How are single-dimension and two-dimension arrays declared and initialized?

(f) What is file? Write a program to create a text file.

4. Answer any *three* of the following questions : 5×3=15

(a) Explain the use of constructor and detractors in C++ with the help of an example.

(b) What are the different types of token available in C++? Explain.

(c) What is library function? Name any three library functions and three preprocessor directives in C++.

(d) Define functions. What are the various methods of parameter passing to the functions?

Paper : GE–1 (C)

(Finite Element Methods)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. (a) Write one advantage of finite element method over finite difference method. 1

(b) State True or False : 1

In residual method, the solution is approximated over the entire region using global trial functions.

(c) Write one difference between least square method and Galerkin method. 2

(d) Write about finite element partition of an interval $[a, b]$. 2

(e) Derive variational formulation of the following BVP : 4

$$-\frac{d^2y}{dx^2} + y = f(x) \text{ in } (0, 1)$$

$$y(0) = 0 \text{ and } y(1) = 0$$

(f) Solve the BVP using the collocation method with two collocation points : 10

$$\frac{d^2y}{dx^2} - y = 0, \quad 0 \le x \le 1$$

$$y(0) = 0, \quad y(1) = 1$$

Or

Describe Ritz variational method.

2. (a) Define the term 'element' in finite element method. 2

(b) Derive the weak form of the BVP : 10

$$-\frac{d}{dx}\left(p(x)\frac{dy}{dx}\right)+q(x)y=f(x),\ a\le x\le b$$

with Dirichlet boundary condition

$$y(a)=A,\quad y(b)=B$$

Or

Solve :

$$-\frac{d^2y}{dx^2}+y=0,\ 0\le x\le 1$$

$$y(0)=0 \text{ and } y(1)=1.$$

using uniform mesh of two linear elements.

3. (a) What is the difference between linear and quadratic elements in finite element method? 2

(b) Solve the following BVP using Ritz method : 10

$$-\frac{d^2y}{dx^2}=f(x),\quad 0\le x\le 1$$

$$y(0)=0,\quad y(1)=0$$

$$f(x)=x$$

Or

Solve

$$-\frac{d^2y}{dx^2}+y=0, \quad 0\le x\le 1$$

$$y'(0)=1, \quad y(1)=2$$

using Ritz method.

4. (a) Write two common types of elements used in two dimensional finite element analysis. 2

(b) Describe finite element model for the Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

on the rectangular domain

$$R=\{(x, y)\in \mathbb{R}^2 : a\le x\le b, c\le x\le d\}$$ 10

Or

Describe rectangular element mesh assembly.

5. (a) Define shape function in finite element method. 2

(b) Derive linear shape functions for the domain [0, 1] using three elements. 10

Or

Derive quadratic shape functions for the domain [0, 1] using two elements.

(Turn Over)

6. Solve :

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$$

$$u(x, 0) = \sin x, \quad 0 \le x \le 1$$
$$u(0, t) = 0$$
$$u(1, t) = 0, \quad t > 0$$ 12

Or

Solve :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = x$$

$$u(x, 0) = 0, \quad 0 \le x \le 1$$
$$u(x, 1) = 0, \quad 0 \le x \le 1$$
$$u(0, y) = 0, \quad 0 \le y \le 1$$
$$u(1, y) = 0, \quad 0 \le y \le 1$$

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