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1 SEM TDC MTMH (CBCS) C 1

2024

(November)

MATHEMATICS

(Core)

Paper : C–1

(Calculus)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Write the domain of definition of the function $\cosh^{-1} x$. 1

(b) Write the necessary condition for the function $f(x)$ to have an extreme value at $x = c$. 1

(c) Find y_n if $y = e^{ax} \cos bx$. 3

Or

If $y = a\cos(\log x) + b\sin(\log x)$, show that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

(d) Find the asymptote of the curve

$$y = \frac{x^2}{x^2 + 1}$$

parallel to x-axis. 3

(e) Evaluate any *one* of the following : 3

(i) $\lim_{x \to 0} \frac{x - \sin x}{x^3}$

(ii) $\lim_{x \to 2} \frac{x^7 - 128}{x^3 - 8}$

(f) Find the range of values of x for which the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave up or concave down. Also determine the points of inflection. 3+1=4

(g) Trace the curve $y = x^3 - 12x - 16$. 5

Or

A manufacturer estimates that when x units of a particular commodity are produced each month, the total cost (in rupees) will be $C(x) = \frac{1}{8}x^2 + 4x + 200$ and all units can be sold at a price of $p(x) = 49 - x$ rupees per unit. Determine the price that corresponds to the maximum profit.

2. Answer any *three* of the following : 5×3=15

(a) Obtain the reduction formula for

$$\int_0^{\pi/2} \sin^n x \, dx$$

(b) Find the volume and area of curved surface of a paraboloid of revolution formed by revolving the parabola $y^2 = 4ax$ about the x-axis and bounded by the section $x = x_1$.

(c) Find the volume and the surface area of the solid generated by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about its base.

(d) Show that

$$I_n = \int_0^\infty \frac{dx}{(1+x^2)^n} = \frac{2n-3}{2n-2} I_{n-1}$$

(e) Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, $x = 1$, $x = 4$ and the x-axis is revolved about the y-axis.

3. (a) Suppose that the axes of any xy-coordinate system are rotated through an angle of $\theta = 45°$ to obtain an $x'y'$-coordinate system. Find the equation of the curve $x^2 - xy + y^2 - 6 = 0$ in $x'y'$-coordinate. 5

(b) Answer any *two* of the following : 4×2=8

(i) Find the arc length of the curve $y = x^{3/2}$ from $(1, 1)$ to $(2, 2\sqrt{2})$.

(ii) Sketch the graph of the ellipse $x^2 + 2y^2 = 4$ showing the foci.

(iii) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y-axis.

(c) Find the new coordinates of the point (2, 4) if the coordinate axes are rotated through an angle $\theta = 30°$. 2

4. (a) If $\vec{r} = \vec{a}\cos\omega t + \vec{b}\sin\omega t$, then show that

$$\frac{d^2\vec{r}}{dt^2} = -\omega^2\vec{r}$$ 2

(b) Prove that

$$\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$$ 2

(c) Answer any *two* of the following : 3×2=6

(i) Find the unit tangent vector at any point on the curve $x = a\cos t$, $y = a\sin t$ and $z = bt$.

(ii) Find $\lim_{t \to 1} [\vec{F}(t) \times \vec{G}(t)]$, where

$$\vec{F}(t) = t\hat{i} + (1-t)\hat{j} + t^2\hat{k}$$

and $\vec{G}(t) = e^t\hat{i} - (3 + e^t)\hat{k}$

(iii) Find

$$\int_0^{\pi} [t\hat{i} + 3\hat{j} - (\sin t)\hat{k}]\, dt$$

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