Total No. of Printed Pages—5

6 SEM TDC DSE MTH (CBCS) 3 (H)

2025

(May)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–3

(Discrete Mathematics)

Full Marks : 80

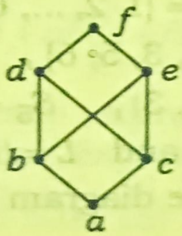
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) Find the least upper bound of the partially ordered set $(\{\ldots, -2, -1, 0, 1\}, \leq)$. 1

   (b) Is the following partially ordered set a lattice? Justify : 2



(c) Let $(L, \leq)$ be a lattice. Show that for any $a, b, c \in L$, $a \wedge (a \vee b) = a$. 3

(d) Define a lattice homomorphism. Let $f : (L_1, \leq_1) \rightarrow (L_2, \leq_2)$ be a lattice homomorphism. Show that $a \leq_1 b \Rightarrow f(a) \leq_2 f(b)$ for any $a, b \in L_1$. 2+2=4

2. Answer any *three* of the following : 5×3=15

(a) Let $(L, \leq)$ be a lattice. Show that

(i) $b \leq c \Rightarrow a \wedge b \leq a \wedge c$, $a, b, c \in L$

(ii) $a \leq b$ and $a \leq c \Rightarrow a \leq b \vee c$, $a, b, c \in L$

(b) Let D_m be the set of all positive divisors of m and $a \mid b$ means a divides b. Show that the lattice $(D_m, \mid)$ is distributive.

(c) Define a chain. Show that every chain $(L, \leq)$ is a distributive lattice.

(d) Let $S_0 = \{1, 2, ..., 6\}$, $S_1 = \{1, 2, ..., 5\}$, $S_2 = \{1, 2, 3, 5, 6\}$, $S_3 = \{1, 2, 3, 5\}$, $S_4 = \{1, 2, 3\}$, $S_5 = \{1, 2\}$, $S_6 = \{1, 3\}$, $S_7 = \{1\}$ and $L = \{S_0, S_1, ..., S_7\}$. Draw the Hasse diagram for the lattice $(L, \subseteq)$.

3. (a) State True or False : 1

In a Boolean algebra B, $(a+b)' = a' \cdot b' \ \forall\ a,\ b \in B$.

(b) Show that a lattice L with 3 elements is not a Boolean algebra. 2

(c) Let $(L, \vee, \wedge)$ be a distributive lattice. Show that for any $a, b, c \in L$ we have $a \wedge b = a \wedge c$ and $a \vee b = a \vee c \Rightarrow b = c$. 3

(d) Show that the diamond lattice M_5 is modular but the pentagon lattice N_5 is not modular. 2+2=4

4. Answer any *three* of the following : 5×3=15

(a) Find a minterm of the variables x, y, z. Obtain the sum of products canonical form of the following Boolean expression in x, y, z :

$$(x+y)z + (y + \bar{x}z)(\overline{x+y})$$

(b) Find a minimal sum-of-products representation of the following Boolean expression using Karnaugh map :

$$abcd + ab\bar{c}\bar{d} + a\bar{b}\bar{c}d + \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}b\bar{c}\bar{d}$$

(c) Find a minimal expression for the following Boolean expression :

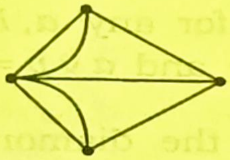
$$\sum(7, 9, 10, 11, 14, 15) \text{ in } a, b, c, d$$

(d) Let B be a Boolean algebra and $a, b \in B$. Show that—

(i) $(a \cdot b)' = a' + b'$

(ii) $a \cdot b = a \Rightarrow a \cdot b' = 0$

5. *(a)* Is the following graph simple? 1



(b) State True or False : 1

An Euler graph is Hamiltonian.

(c) Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$. 3

(d) Write three properties of an adjacency matrix of a graph. 3

(e) Let G be a graph with $n \geq 2$ vertices and G has exactly two odd vertices. Show that there exists a path between the two odd vertices in G. 3

(f) Show that the maximum number of edges in a complete bipartite graph of n vertices is $\left[\frac{n^2}{4}\right]$. 4

6. Answer any *three* of the following : 5×3=15

(a) Let $n \geq 3$ be odd. Show that the number of edge-disjoint Hamiltonian circuits in K_n is $\frac{n-1}{2}$.

(b) Show that a connected graph G is Euler iff G can be decomposed into circuits.

(c) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

(d) Prove that a simple graph with n vertices is connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges.

(e) Write a short note on Dijkstra's algorithm.

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