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**6 SEM TDC STS H (CBCS) C 14 (N/O)**

**2025**

( May )

**STATISTICS**

( Core )

Paper : C-14

**( Multivariate Analysis and Non-Parametric Methods )**

*The figures in the margin indicate full marks  
for the questions*

( New Course )

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the following alternatives : 1×6=6

(a) If the joint distribution of the variables  $X$  and  $Y$  is BVN  $(0, 0, 1, 1, \rho)$ , then the correlation coefficient between  $X^2$  and  $Y^2$  is equal to

(i) 1

(ii) -1

(iii)  $\rho^2$

(iv) 0

(b) A measure of linear association of a variable say  $X_1$  with a number of other variables  $X_2, X_3, \dots, X_k$  is known as

- (i) partial correlation
- (ii) multiple correlation
- (iii) simple correlation
- (iv) autocorrelation

(c) Principal component analysis is meant for

- (i) reducing the number of observed variables
- (ii) increasing the number of variables
- (iii) reducing the correlation between variables
- (iv) reducing the variance among variables

(d) Factor analysis is a technique in which the observed variables are expressed in terms of a

- (i) factor
- (ii) linear combination of factors
- (iii) correlation
- (iv) distance



- (e) A sequence of symbols shows lack of randomness if there are
- (i) too many runs
  - (ii) too few runs
  - (iii) Both (i) and (ii)
  - (iv) Neither (i) nor (ii)
- (f) Kolmogorov test is based on the theorem
- (i) N. V. Smirnov
  - (ii) A. N. Kolmogorov
  - (iii) Kolmogorov-Smirnov
  - (iv) Glivenko-Cantelli

2. Answer the following questions in brief :  $2 \times 6 = 12$

- (a) State the properties of random vector.
- (b) Define the probability density function of multivariate normal distribution.
- (c) State the features of principal component analysis.
- (d) State the assumptions underlying discriminant analysis.
- (e) What are the assumptions of non-parametric test?
- (f) How do you solve the problem of zero differences in sign test?

3. (a) State the properties of bivariate normal distribution.

Let the joint p.d.f. of  $X$  and  $Y$  be

$$f(x, y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2(1-\rho^2)} \left[ \left( \frac{x-\mu_1}{\sigma_1} \right)^2 - 2\rho \left( \frac{x-\mu_1}{\sigma_1} \right) \left( \frac{y-\mu_2}{\sigma_2} \right) + \left( \frac{y-\mu_2}{\sigma_2} \right)^2 \right] \right\}$$

where  $-\infty < x < \infty$ ,  $-\infty < y < \infty$ ,  $-1 < \rho < 1$ .

Find—

- (i) the conditional distribution of  $X | Y = y$ ;

- (ii)  $P(5 < X < 9 | Y = 6)$ . Given that

$$\mu_1 = 3, \mu_2 = 4, \sigma_1^2 = 16, \sigma_2^2 = 25,$$

$$\rho = 0.8, \phi(1.97) = 0.47 \quad \text{and}$$

$$\phi(0.3) = 0.11791.$$

$$2+3+2=7$$

Or

- (b) How do you express and organize multivariate data?

Let

$$X = \begin{bmatrix} 10 & 100 \\ 12 & 110 \\ 11 & 105 \end{bmatrix}$$

Calculate the mean vector and variance-covariance matrix.

$$3+4=7$$



4. Answer any *two* from the following questions :

6×2=12

- (a) If  $X \sim N_p(\underline{\mu}, \Sigma)$ , then show that the components of  $X$  will be jointly independent if and only if the covariance of  $X_j$  and  $X_k$  ( $j \neq k = 1, 2, \dots, p$ ) is zero. 6
- (b) State and prove the reproductive property of multivariate normal distribution. 1+5=6
- (c) Explain the concept of partial correlation coefficient with examples. Show that partial correlation coefficient is the geometric mean between the regression coefficient. 3+3=6
- (d) Define multiple correlation with example. Discuss the different properties of multiple correlation. 3+3=6

5. (a) For what purposes discriminant analysis is used? How to obtain discriminant scores? Explicate Fisher's linear discriminant function. 2+2+5=9

Or

- (b) What are the uses of principal component analysis? Distinguish between principal component and factor analysis. Deliberate the basic approach of factor analysis. 2+2+5=9

6. (a) What do you mean by empirical distribution function? Write some applications of Kolmogorov-Smirnov test. Explicate Mann-Whitney U-test for testing the identicalness of two populations.  $2+2+5=9$

Or

- (b) State the assumptions of Kruskal Wallis test. Discuss the Kruskal Wallis method of analysis for one-way classification of data.  $3+6=9$



( 7 )

( Old Course )

Full Marks : 50

Pass Marks : 20

Time : 2 hours

1. Choose the correct answer from the following alternatives :

1×5=5

(a) If the joint distribution of the variables  $X$  and  $Y$  is BVN  $(0, 0, 1, 1, \rho)$ , then the correlation coefficient between  $X^2$  and  $Y^2$  is equal to

(i) 1

(ii) -1

(iii)  $\rho^2$

(iv) 0

(b) A measure of linear association of a variable say  $X_1$  with a number of other variables  $X_2, X_3, \dots, X_k$  is known as

(i) partial correlation

(ii) multiple correlation

(iii) simple correlation

(iv) autocorrelation

- (c) Principal component analysis is meant for
- (i) reducing the number of observed variables
  - (ii) increasing the number of variables
  - (iii) reducing the correlation between variables
  - (iv) reducing the variance among variables
- (d) A sequence of symbols shows lack of randomness if there are
- (i) too many runs
  - (ii) too few runs
  - (iii) Both (i) and (ii)
  - (iv) Neither (i) nor (ii)
- (e) Kolmogorov test is based on the theorem
- (i) N. V. Smirnov
  - (ii) A. N. Kolmogorov
  - (iii) Kolmogorov-Smirnov
  - (iv) Glivenko-Cantelli



2. Answer the following questions in brief :  $2 \times 5 = 10$

- (a) State the properties of random vector.
- (b) Define the probability density function of multivariate normal distribution.
- (c) State the features of principal component analysis.
- (d) What are the assumptions of non-parametric test?
- (e) How do you solve the problem of zero differences in sign test?

3. (a) State the properties of bivariate normal distribution.

Let the joint p.d.f. of  $X$  and  $Y$  be

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where  $-\infty < x < \infty$ ,  $-\infty < y < \infty$ ,  $-1 < \rho < 1$ .

Find—

- (i) the conditional distribution of  $X|Y = y$ ;

(ii)  $P(5 < X < 9 | Y = 6)$ . Given that

$$\mu_1 = 3, \mu_2 = 4, \sigma_1^2 = 16, \sigma_2^2 = 25,$$

$$\rho = 0.8, \quad \phi(1.97) = 0.47 \quad \text{and}$$

$$\phi(0.3) = 0.11791. \quad 2+3+2=7$$

Or

(b) How do you express and organize multivariate data?

Let

$$X = \begin{bmatrix} 10 & 100 \\ 12 & 110 \\ 11 & 105 \end{bmatrix}$$

Calculate the mean vector and variance-covariance matrix.

$$3+4=7$$

4. Answer any *two* from the following questions :

$$6 \times 2 = 12$$

(a) If  $X \sim N_p(\underline{\mu}, \Sigma)$ , then show that the components of  $X$  will be jointly independent if and only if the covariance of  $X_j$  and  $X_k$  ( $j \neq k = 1, 2, \dots, p$ ) is zero.

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(b) State and prove the reproductive property of multivariate normal distribution.

$$1+5=6$$



- (c) Explain the concept of partial correlation coefficient with examples. Show that partial correlation coefficient is the geometric mean between the regression coefficient.  $3+3=6$
- (d) Define multiple correlation with example. Discuss the different properties of multiple correlation.  $3+3=6$
5. (a) For what purposes discriminant analysis is used? How to obtain discriminant scores? Explicate Fisher's linear discriminant function.  $2+2+4=8$

Or

- (b) What are the uses of principal component analysis? Distinguish between principal component and factor analysis. Deliberate the basic approach of factor analysis.  $2+2+4=8$
6. (a) What do you mean by empirical distribution function? Write some applications of Kolmogorov-Smirnov test. Explicate Mann-Whitney U-test for testing the identicalness of two populations.  $2+2+4=8$

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- (b) State the assumptions of Kruskal-Wallis test. Discuss the Kruskal-Wallis method of analysis for one-way classification of data.

3+5=8

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