**5 SEM TDC MTH M 1**

**2019**

( November )

MATHEMATICS

( Major )

Course : 501

**( Logic and Combinatorics, and Analysis—III )**

*Full Marks* : 80

*Pass Marks* : 32/24

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

GROUP—A

**( Logic and Combinatorics )**

( *Marks* : 35 )

**1.** *(a)* *(i)* If $P \leftrightarrow Q$ is true, what can be said about the truth value of $P \vee {\sim} Q$? 1

(ii) Is there any sentence $P$ such that $P \wedge \sim P$ is a tautology? 1

(b) Construct the truth table of the following statement : 4

$$P \to \sim (Q \wedge R)$$

*Or*

Using rules of inferences, prove that $W \vee P \to I$, $I \to C \vee S$, $S \to U$, $\sim C \wedge \sim U \vDash \sim W$.

(c) Prove that $A \vDash B$ iff $A \to B$. 4

*Or*

Prove that every truth function can be generated by $\sim$, $\wedge$ and $\vee$ only.

2. (a) Define a term. 1

(b) Translate the following in symbols : 1×2=2

(i) All judges are lawyers.

(ii) Some reals are not rational.

(c) Find a formal derivation of

$$A \vee B,\ A \to C,\ B \to D \vDash C \vee D$$ 3

(d) Derive mathematically any *one* of the following : 4

(i) Every member of the committee is wealthy and a republican. Some committee members are old. Therefore, there are some old republicans.

(ii) If $A$ wins, then either $B$ or $C$ will place. If $B$ places, then $A$ will not win. If $D$ places then $C$ will not. Therefore, if $A$ wins, $D$ will not place.

3. (a) State Pascal's identity. 1

(b) How many ways are there to place 10 indistinguishable balls into 8 distinguishable bins? 2

(c) Show that $R(4, 4) = 18$. 4

*Or*

State Bell number. Prove that

$$B_n = \sum_{k=0}^{n-1} C(n-1, k) B_k$$

**4.** *(a)* State the pigeonhole principle. 1

*(b)* Use generating functions to find the number of ways to select $r$ objects of $n$ different kinds if we must select at least one object of each kind. 4

*Or*

Find the generating function of the sequence 1, $a$, $a^2$, ... where $a$ is a fixed constant.

*(c)* Find the number of positive integers less than 601 that are not divisible by 3 or 5 or 7. 3

GROUP—B

**[ Analysis—III (Complex Analysis) ]**

( *Marks* : 45 )

**5.** *(a)* Define isolated singularity of a complex function $f(z)$. 1

*(b)* What do you mean by conjugate harmonic functions? 1

*(c)* Prove that an analytic function with constant real part is constant. 3

(d) Derive the polar form of Cauchy-Riemann equations. 5

*Or*

Show that $U = \frac{1}{2}\log(x^2 + y^2)$ is harmonic and find its harmonic conjugate.

6. (a) Define a contour. 1

(b) Verify Cauchy's theorem for the function $f(z) = z^3 - iz^2 - 5z + 2i$ if the path of the circle given by $|z-1| = 2$. 4

(c) State and prove Cauchy's integral formula. 5

(d) Answer any *one* of the following : 4

(i) Evaluate :

$$\int_C \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)} dz$$

(ii) If the function $f(z)$ is analytic for finite values of $z$, and is bounded, then show that $f(z)$ is constant.

7. *(a)* State Taylor's series. When does it reduce to Maclaurin's series? 1+1=2

*(b)* If the function $f(z)$ is analytic and one valued in $|z-a|<R$, prove that, when $0<r<R$

$$f'(a)=\frac{1}{\pi r}\int_0^{2\pi} P(\theta)e^{-i\theta}\,d\theta$$

where $P(\theta)$ is the real part of $f(a+re^{i\theta})$. 6

*Or*

Obtain the Taylor and Laurent series which represents the function

$$\frac{z^2-1}{(z+2)(z+3)}$$

in the regions *(i)* $|z|<2$ and *(ii)* $2<|z|<3$.

8. *(a)* Define a simple zero of an analytic function. 1

*(b)* Find the singularities of $\tan(\frac{1}{z})$ at $z=0$. 2

*(c)* Evaluate any *two* of the following : 5×2=10

*(i)* $\int_0^\infty \frac{dx}{x^4 + a^4}$ if $a > 0$

*(ii)* $\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta}$

*(iii)* $\int_{-\infty}^\infty \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$

*(iv)* $\int_0^\pi \frac{a\, d\theta}{a^2 + \sin^2\theta}$ where $a > 0$

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