5 SEM TDC MTH M 3

2019

(November)

MATHEMATICS

(Major)

Course : 503

(Fluid Mechanics)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) Hydrodynamics

(*Marks* : 35)

1. *(a)* Choose the correct answer : 1

D/Dt is known as

(i) static differential operator

(ii) partial differential operator

(iii) total differential operator

(iv) differentiation following the motion

(b) Define real fluid with example. 2

(c) Fill in the blank : 1

If ϕ be the velocity potential, then it satisfies _____ equation.

(d) Define velocity of a fluid particle. 1

(e) Determine the acceleration of a fluid particle from the following flow-field : 5

$$q = \hat{i}(Axy^2t) + \hat{j}(Bx^2yt) + \hat{k}(Cxyz)$$

(f) Deduce the equation of continuity. 5

Or

If $u = \dfrac{ax - by}{x^2 + y^2}$, $v = \dfrac{ay + bx}{x^2 + y^2}$, $w = 0$,

investigate the nature of the motion of the liquid.

2. (a) Write down the Euler's equation of motion. 1

(b) Deduce the equation of motion for impulsive forces. 6

(c) What is meant by circulation? Write the statement of Kelvin's circulation theorem.

(d) Write the Bernoulli's equation for steady and unsteady fluid motions.

3. *(a)* Choose the correct answer : 1

Let C be the closed curve and T be the circulation, then

(i) $T = \int q \cdot dr$

(ii) $T = \int q \times dr$

(iii) $T = \int |q| dr$

(iv) None of the above

(b) Show that kinetic energy of liquid is given by

$$-\frac{1}{2}\rho \int_S \phi \frac{\partial \phi}{\partial n} dS$$

where ϕ is the single-valued velocity potential over the surface S, ρ be the density of the liquid. 7

Or

Prove that there cannot be two different forms of irrotational motion for a given confined mass of incompressible inviscid liquid whose boundaries are subject to the given impulses.

(B) Hydrostatics

(*Marks* : 45)

4. *(a)* Define specific gravity of a substance. 1

(b) Write True or False : 1

If a triangle is immersed in a homogeneous liquid, then the sum of the pressures at the vertices is equal to three times the pressure at the centroid of the triangle.

(c) Write down the differential equation of fluid pressure. Find the necessary condition that must be satisfied by a given system of external forces, so that the fluid may maintain equilibrium.

1+3=4

5. *(a)* A small uniform tube is bent into the form of a circle whose plane is vertical. Equal quantities of two fluids of densities ρ and σ fill half the tube. Show that the radius passing through the common surface makes with the vertical an angle θ given by

$$\tan\theta = \frac{\rho - \sigma}{\rho + \sigma}$$

6

Or

A fine circular tube in the vertical plane contains a column of liquid of density δ, which subtends a right angle at the centre, and a column of density δ' subtending an angle α. Prove that the radius through the common surface makes with the vertical an angle

$$\tan^{-1}\frac{\delta-\delta'+\delta'\cos\alpha}{\delta+\delta'\sin\alpha}$$

(b) Find the pressure at a point in the lower layer of two given heavy homogeneous liquids which do not mix. 5

6. *(a)* Define centre of pressure. 1

(b) Find the centre of pressure of a parallelogram immersed in a homogeneous liquid with one side in the surface. 5

7. *(a)* A semi-circular lamina is immersed in a liquid with diameter in the surface. Find the depth of the centre of pressure. 5

Or

A triangle ABC is immersed in a liquid, its plane being vertical and the side AB in the surface; if O be the centre of the circumscribed circle of ABC, prove that

$$\frac{\text{Pressure on the } \Delta AOC}{\text{Pressure on the } \Delta OCB} = \frac{\sin 2B}{\sin 2A}$$

(b) A closed cylindrical vessel with hemispherical ends is filled with water and placed with its axis horizontal. Find the resultant thrust on each of the ends and determine its line of action. 5

Or

A solid hemisphere of radius a is placed with its centre at a distance h below the surface of water and has its plane face vertical. Find the horizontal thrust on the curved surface. Find also the resultant thrust on it.

8. *(a)* Write the condition of equilibrium if a body floats in two liquids which do not mix. 2

(b) Define surface of floatation and curves of floatation. 2

(c) What are the different types of equilibrium in respect of metacentre with respect to the positions of the centre of gravity? 3

(d) A uniform rod of length $2l$ can turn freely about one end which is fixed at height $h(<2l)$ above the surface of the liquid. If the densities of the rod and the liquid be ρ and σ respectively, show that the rod can rest *either* in a vertical position *or* inclined at an angle θ to the vertical such that

$$\cos\theta = \frac{h}{2l}\sqrt{\frac{\sigma}{\sigma-\rho}}$$ 5

Or

If W_1, W_2, W_3 be the apparent weights of a given body in fluids whose specific gravities are S_1, S_2, S_3, then prove that

$$W_1(S_2 - S_3) + W_2(S_3 - S_1) + W_3(S_1 - S_2) = 0$$

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