5 SEM TDC MTH M 4

2019

(November)

MATHEMATICS

(Major)

Course : 504

(Mechanics and Integral Transform)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mechanics)

(a) : Statics

(*Marks* : 25)

1. *(a)* Write the definition of wrench. 1

(b) Choose the correct one : 1

(i) Screw is a curve.

(ii) Screw is a couple.

(iii) Screw is a force.

(iv) Screw is a definite straight line associated with a definite pitch.

(c) Find the equations of the central axis of a system of forces acting on a rigid body. 8

Or

Find the necessary and sufficient conditions for equilibrium of a rigid body.

2. (a) Write the radius of curvature at any point of a catenary. 1

(b) Write two forces which can be omitted while constructing equation of virtual work. 2

(c) Deduce the Cartesian equation of common catenary. 5

Or

Four uniform rods are joined to form a rectangle *ABCD*. *AB* is fixed in a vertical position with *A* uppermost, and the rectangle is kept in shape by a string joining *AC*. Find the tension of the string.

(d) State and prove the principle of virtual work for a system of coplanar forces acting at different points of a rigid body. 7

Or

A hemisphere rests in equilibrium on a sphere of equal radius; show that the equilibrium is unstable when the curved surface of the hemisphere and stable when the flat surface of the hemisphere rests on the sphere.

(b) : Dynamics

(*Marks* : 25)

3. (a) Derive the equation of simple harmonic motion. 4

Or

Find the radial component of velocity of a particle.

(b) If the radial and transverse velocities of a particle are always proportional to each other, then show that the path is an equiangular spiral. 4

4. *(a)* Write the equation of central orbit in pedal form. 1

(b) Choose the correct answer for the following : 1

Resisting force

(i) acts along the direction of motion

(ii) is non-conservative

(iii) is conservative

(iv) obeys principle of conservation of energy

(c) A particle describes the curve $r^n = a^n \cos n\theta$ under a force F to the pole. Find the law of force. 5

Or

A particle falls under gravity, supposed constant in a resisting medium whose resistance varies as the square of the velocity. Find the motion of the particle, if it starts from rest.

5. *(a)* Define radius of gyration. 1

(b) State and prove the theorem of perpendicular axes. 5

Or

Prove that the reversed effective forces acting on each particle of the body and the external forces of the system are in equilibrium.

(c) Find the moment of inertia of a rectangular lamina about a line through its centre and parallel to one of its edges. 4

GROUP—B

(Integral Transform)

(*Marks* : 30)

6. *(a)* Write the values of the following : 1×3=3

(i) $L\{t^2\}$

(ii) $L\{\sin^2 t\}$

(iii) $L\{e^{2t}\}$

(b) Find $L\{te^{3t}\}$. 2

(c) Find $L\{t^2 \cos^2 t\}$. 3

Or

Show that, if $L\{F(t)\} = f(s)$, then

$$L\{F(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right)$$

7. *(a)* Write the value of

$$L^{-1}\left\{\frac{1}{s-1}\right\}$$ 1

(b) Evaluate : 3

$$L^{-1}\left\{\frac{e^{-2s}}{(s-2)^2}\right\}$$

(c) Evaluate : 4

$$L^{-1}\left\{\log\frac{s+6}{s+3}\right\}$$

Or

Evaluate $L^{-1}\left\{\frac{s}{(s^2+4)^2}\right\}$.

8. *(a)* Write the value of $L\left\{\frac{\partial^2 y}{\partial x^2}\right\}$. 1

(b) Solve any *two* of the following : 4×2=8

(i) $\frac{d^2y}{dt^2} + y = \cos t,\ y(0) = 0,\ y'(0) = 0$

(ii) $\frac{d^2y}{dt^2} + 9y = 6\cos 3t,\ y(0) = 2,\ y'(0) = 0$

(iii) $\frac{d^2y}{dt^2} + y = t\cos 2t,\ t > 0,\ y(0) = 0,\ y'(0) = 0$

(c) Solve the following : 5

$$\frac{dx}{dt} + \frac{dy}{dt} = t$$

$$\frac{d^2x}{dt^2} - y = e^{-t},\ x(0) = 0,\ y(0) = 0,\ x'(0) = 0$$

Or

Find the bounded solution of

$$\frac{\partial y}{\partial x} = 2\frac{\partial y}{\partial t} + y, \text{ if } y(x, 0) = 6e^{-3x}$$

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