5 SEM TDC MTH M 3

2018

(November)

MATHEMATICS

(Major)

Course : 503

(Fluid Mechanics)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) Hydrodynamics

(*Marks* : 35)

1. *(a)* Write the relation between material, local and convective derivatives. 1

(b) Define velocity potential. Under what condition, the flow is known as the potential kind? 2+1=3

(c) The differential equation of streamline is

(i) $\vec{q} \times dr = 0$

(ii) $\vec{q} \cdot dr = 0$

(iii) $r \cdot dq = 0$

(iv) None of the above 1

(Choose the correct one)

(d) The velocity components in three-dimensional flow for an incompressible fluid are $(2x, -y, -z)$. Is it a possible field? Determine the equation of streamline passing through (1, 1, 1). 2+3=5

(e) Express the acceleration of a fluid particle in Cartesian coordinate. 5

2. (a) Write the equation of motion of an incompressible fluid under impulsive force. 1

(b) Define flow and circulation. 4

(c) Deduce Euler's equation of motion. 7

Or

State and prove Kelvin's circulation theorem.

3. *(a)* State Green's theorem. 2

(b) Liquid is contained between two parallel planes; the free surface is a circular cylinder of radius a, whose axis is perpendicular to the planes. All the liquids within a concentric circular cylinder of radius b are suddenly annihilated. Prove that, if π be the pressure at the outer surface, the initial pressure at any point of the liquid at distance r from the centre, is

$$\pi\left(\frac{\log r - \log b}{\log a - \log b}\right)$$ 6

Or

A velocity field is given by

$$q = \left(\frac{-iy + jx}{x^2 + y^2}\right)$$

Calculate the circulation round a square with its corners at (1, 0), (2, 0), (2, 1) and (1, 1).

(B) Hydrostatics

(*Marks* : 45)

4. *(a)* Fill in the blanks : 1×2=2

(i) If W be the weight of a volume V of a substance whose specific gravity is s and w be the weight of a unit volume of the standard substance, then $W =$ _____ .

(ii) The rate of increase of the pressure in any direction is equal to the product of the _____ and the component of external forces in that direction.

(b) What is surface of equi-pressure? Write down its mathematical form for a field in equilibrium. What will be its shape when the fluid is at rest under gravitational force? 1+2+1=4

(c) In a uniform circular tube, two liquids are placed so as to subtend 90° each at the centre. If the diameter joining the two free surfaces be inclined at 60° to the vertical, prove that the densities of the two liquids are as $\frac{\sqrt{3}+1}{\sqrt{3}-1}$. 5

(*Continued*)

(d) Prove that pressure at a point of a fluid at rest is same in all directions. 6

Or

Show that the specific gravity of a mixture of n liquids is greater when equal volumes are taken than when equal weights are taken, assuming no change in volume as the result of mixing.

5. (a) Write True or False : 1

The principle of Archimedes is the result to find the resultant thrust on a solid immersed in a fluid.

(b) What is the centre of pressure for a plane surface immersed in a liquid? Is it a single point? Justify. 2+1+1=4

(c) Find the centre of pressure of a triangular area immersed in a liquid with its vertex in the surface and base horizontal. 6

Or

Prove that the horizontal line through the centre of pressure of a rectangle immersed in a liquid with one side in the surface, divides the rectangle in two parts, the fluid pressure on which are in the ratio 4 : 5.

(d) A hemisphere bowl is filled with liquid and placed in an inverted position in contact with a horizontal table and no water comes out. Show that the resultant vertical thrust on its curved surface is one-third of the thrust on the table. 5

Or

A conical wineglass is filled with water and placed in an inverted position upon a table. Show that the resultant vertical thrust of the water on the glass is two-thirds that on the table.

6. *(a)* State the conditions of equilibrium of a body freely floating in a liquid. 2

(b) Define free surface and effective surface of a liquid. 2

(c) Define metacentre. Mention the state of equilibrium of the floating body when the metacentre lies below the centre of gravity. 2+1=3

(d) A rod of small cross-section and of density ρ has a small portion of metal of weight $\frac{1}{n}$th that of the rod attached to one extremity. Prove that the rod will float at any inclination in a liquid of density σ if $(n+1)^2\rho = n^2\sigma$. 5

Or

A thin metallic circular cylinder contains water to a depth h and floats in water with its axis vertical, immersed to a depth h'. Show that the vertical position is stable if the height of the centre of gravity of the cylinder above its base is less than $\frac{1}{2}(h+h')$.

★★★