**5 SEM TDC MTH M 4**

**2018**

( November )

# MATHEMATICS

( Major )

Course : 504

**( Mechanics and Integral Transform )**

*Full Marks* : 80

*Pass Marks* : 32/24

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

## GROUP—A

**( Mechanics )**

**(a) : Statics**

( *Marks* : 25 )

**1.** *(a)* Define moment of a force. 1

*(b)* Write when a system is called equipollent to zero. 1

*(c)* Write the quantities which are invariants for any given system of forces. 1

(d) Find the equations of the central axis of a system of forces acting on a rigid body. 7

*Or*

Find the null point of the plane $lx + my + nz = 1$. 7

2. (a) Define virtual work. 2

(b) Deduce the intrinsic equation of common catenary. 7

*Or*

A regular hexagon $ABCDEF$ consists of six equal uniform rods, each of weight $w$, freely jointed together. The hexagon rests in a vertical plane and $AB$ is in contact with a horizontal table, if $c$ and $F$ be connected by a light string, prove that its tension is $\sqrt{3}\,w$. 7

(c) Discuss the conditions of stability for a body with one degree of freedom. 6

*Or*

In a common catenary, show that,

(i) $s = c\sinh\frac{x}{c}$

(ii) $y = c\sec\psi$

(iii) $T = wy$ 6

**(b) : Dynamics**

( *Marks* : 25 )

3\. (a) Write the value of $\frac{d}{dt}(\hat{h})$. 1

(b) Find the radial and transverse components of acceleration. 7

*Or*

Let a particle moves in a plane curve, so that its tangential and normal accelerations are equal and the angular velocity of the tangent is constant. Find the curve. 7

4\. (a) Resisting force

(i) is conservative

(ii) is non-conservative

(iii) acts along the direction of motion

(iv) None of the above

( Choose the correct option ) 1

(b) A particle describes a circle, pole on its circumference, under a force $P$ to the pole. Find the law of force. 6

*Or*

A particle is projected upwards under gravity, supposed constant, in a resisting medium whose resistance varies as the square of the velocity. Find the motion. 6

**5.** *(a)* State the principle of d'Alembert. 1

*(b)* Describe momental ellipsoid. 3

*(c)* Find the moment of inertia of a uniform triangular lamina about one side. 6

*Or*

State and prove the theorem of parallel axes of moment of inertia. 6

## GROUP—B

**( Integral Transform )**

( *Marks* : 30 )

6. (a) Write the values of the following : 1+1+1=3

(i) $L\{t^{\frac{3}{2}}\}$

(ii) $L\{\sin 2t\}$

(iii) $L\{e^{iat}\}$

(b) Find $L\{t \sin 4t\}$. 2

(c) Find $L\{te^{2t} \sin 3t\}$. 3

*Or*

If $f(s) = L\{F(t)\}$, then prove that

$$L\left\{\frac{d^n F(t)}{dt^n}\right\} = s^n f(s) - s^{n-1}F(0)$$

$$-s^{n-2}F'(0) - \cdots - sF^{(n-2)}(0) - F^{(n-1)}(0)$$ 3

7. (a) Write the value of $L^{-1}\left\{\frac{1}{s^2}\right\}$. 1

*( Turn Over )*

(b) Find $L^{-1}\left\{\frac{s+4}{s^2+8s+25}\right\}$. 2

(c) Find (any *one*) : 2

(i) $L^{-1}\left\{\frac{s}{(s+3)^{\frac{3}{2}}}\right\}$

(ii) $L^{-1}\left\{\frac{1}{(s+a)(s+b)}\right\}$

(d) Find $L^{-1}\left\{\frac{s}{(s^2+2)^2}\right\}$.

*Or*

If $L^{-1}\{f(s)\} = F(t)$, then show that

$$L^{-1}\{f(s-a)\} = e^{at} F(t)$$ 3

8. (a) Write the value of $L\left\{\frac{\partial y}{\partial t}\right\}$. 1

(b) Solve (any *two*) : 4×2=8

(i) $\frac{d^2y}{dt^2}+25y = 10\cos 5t,\ y(0)=2,\ y'(0)=0$

(ii) $2\frac{d^2y}{dt^2}+5\frac{dy}{dt}+2y = e^{-2t},\ y(0)=1,\ y'(0)=1$

(iii) $\frac{d^2y}{dt^2}+y = 2,\ y(0)=3,\ y'(0)=1$

(c) Solve : 5

$$\frac{dx}{dt} - y = e^t, \ \frac{dy}{dt} + x = \sin t, \ x(0) = 1, \ y(0) = 0$$

*Or*

Find the bounded solution of $\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}$, $x > 0$, $t > 0$ and $y(0, t) = 1$, $y(x, 0) = 0$. 5

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