5 SEM TDC MTH M 1

2017

(November)

MATHEMATICS

(Major)

Course : 501

(Logic and Combinatorics, and Analysis—III)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) Logic and Combinatorics

(*Marks* : 35)

1. (*a*) (*i*) What do you mean by truth value of a proposition? 1

(*ii*) State the law of syllogism. 1

(*b*) (*i*) Write down the contrapositive statement of $p \to q$. 1

(*ii*) If the value of $p \to q$ is *T*; what can be said about the value of $\sim p \wedge q \leftrightarrow p \vee q$? 2

(c) (i) Express the statement $(p \vee \sim q) \to p \wedge r$ in terms of $\vee$ and $\sim$ only. 1

(ii) Prove that every truth function can be generated by $\sim$, $\wedge$ and $\vee$ only. 4

Or

Prove that if $\models A$ and $\models A \to B$, then $\models B$.

2. (a) Define a term. 1

(b) Translate the following in symbols : 1×2=2

(i) Some rationals are real.

(ii) All women who are lawyers admire some judge.

(c) Find a formal derivation of

$$A \to (B \to C),\ \sim D \vee A,\ B \models D \to C$$ 3

(d) Prove that $\forall x (P(x) \to S(x))$ is the consequence of the following premises : 4

(i) $\forall x (P(x) \to \mathbb{Q}(x))$

(ii) $\forall x (\mathbb{Q}(x) \to S(x))$

Or

Derive mathematically the following (any *one*) :

(i) Every member of the committee is wealthy and a republican. Some committee members are old. Therefore, there are some old republicans.

(ii) All rational numbers are real numbers. Some rationals are integers. Therefore, some real numbers are integers.

3. (a) State multinational theorem. 1

(b) In an election, the number of candidates is one more than the number of vacancies. If a voter can vote in 30 different ways, find the number of candidates. 2

Or

Find the coefficient of $x^3y^3z^2$ in $(2x-3y+5z)^8$.

(c) State and prove the principle of inclusion-exclusion. 4

Or

Find the number of solutions in integers of the equation $a+b+c+d=17$, where $1 \le a \le 3$, $2 \le b \le 4$, $3 \le c \le 5$, $4 \le d \le 6$.

4. *(a)* State the pigeonhole principle. 1

(b) Show that in any set of eleven integers, there are two whose difference is divisible by 10. 3

(c) Find the binomial and exponential generating functions for the sequence 2, 2, 2, ... 4

Or

Find the number of solutions of $e_1 + e_2 + e_3 = 17$, where e_1, e_2 and e_3 are non-negative integers with $2 \le e_1 \le 5$, $3 \le e_2 \le 6$, $4 \le e_3 \le 7$.

(B) Analysis—III (Complex Analysis)

(*Marks* : 45)

5. *(a)* What do you mean by a multiple point? 1

(b) Derive the polar form of Cauchy-Riemann equation. 3

(c) Prove that $u = y^3 - 3x^2 y$ is a harmonic function. Determine its harmonic conjugate and find the corresponding analytic function $f(z)$ in terms of z. 6

Or

If $u + v = \dfrac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z.

6. *(a)* Define Jordan arc. 1

(b) Evaluate

$$\int_C (z^2 + 3z + 2)\, dz$$

where C is the arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ between the points $(0, 0)$ and $(\pi a, 2a)$. 4

(c) State and prove Cauchy's integral theorem. 5

(d) Answer the following (any *one*) : 4

(i) Evaluate

$$\int_C \frac{e^{3z}}{z+i}\,dz$$

where C is the circle $|z+1+i|=2$.

(ii) Evaluate

$$\int_C \frac{z^2-4}{z(z^2+9)}\,dz$$

where C is the circle $|z|=1$.

7. (a) State and prove Taylor's series. 1+5=6

(b) Expand

$$f(z)=\frac{z^2-1}{(z+2)(z+3)}$$

where $|z|>2$. 2

8. (a) Define essential singularity of an analytic function $f(z)$. 1

(b) Discuss the singularity of

$$f(z)=\frac{z^2+4}{e^z}$$

at $z=\infty$. 2

(c) Evaluate the following (any *two*) : 5×2=10

(i) $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$

(ii) $\int_0^{\pi} \frac{\cos 2\theta}{1-2a\cos\theta+a^2} d\theta$

(iii) $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^3}$

(iv) $\int_0^{\infty} \frac{\cos mx}{a^2+x^2} dx; \; m \geq 0$

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