5 SEM TDC MTH M 3

2017

(November)

MATHEMATICS

(Major)

Course : 503

(Fluid Mechanics)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) Hydrodynamics

(*Marks* : 35)

1. *(a)* Write the equation of streamline. 1

(b) Choose the correct alternative : 1

If the motion is irrotational, then

(i) $\vec{\omega} = \frac{1}{2}\text{curl}\,\vec{q} = \vec{0}$

(ii) $\vec{\omega} = \text{curl}\,\vec{q} = \vec{0}$

(iii) $\vec{\omega} = \text{div}\,\vec{q} = \vec{0}$

(iv) None of the above

(c) Why is equation of continuity called equation of conservation of mass? 1

(d) Define vorticity vector. 2

(e) Find the paths of the particle, when

$$u = \frac{x}{1+t},\ v = \frac{y}{1+t},\ w = \frac{z}{1+t}$$ 3

(f) Determine the equation of continuity by vector approach for incompressible fluid. 7

Or

Determine whether the motion specified by $\vec{q} = \frac{A(x\hat{j} - y\hat{i})}{x^2 + y^2}$, ($A$ = constant) is a possible motion, for an incompressible fluid. If so, determine the streamlines.

2. (a) Write down the Euler's equation of motion in vector form. 1

(b) Deduce the equation of motion under impulsive forces. 5

(c) State and prove Bernoulli's equation of unsteady and irrotational flows. 6

Or

A stream in a horizontal pipe, after passing a contraction in the pipe at which its sectional area is A is delivered at atmospheric pressure at a place, where the sectional area is B. Show that if a side tube is connected with the pipe at the former place, water will be sucked up through it into the pipe from a reservoir at a depth $\frac{s^2}{2g}\left(\frac{1}{A^2}-\frac{1}{B^2}\right)$ below the pipe, s being the delivery per second.

3. *(a)* Define acyclic irrotational motion. 1

(b) State True or False : 1

In irrotational motion, the velocity cannot be a maximum in the interior of the fluid.

(c) State and prove Kelvin's minimum energy theorem. 6

Or

If a region lying wholly in a liquid be bounded by a spherical surface, then prove that the mean value of the velocity potential over the surface is equal to its value at the centre of the sphere.

(B) Hydrostatics

(*Marks* : 45)

4. (*a*) Define fluid pressure. 1

(*b*) What is hydrostatic paradox? 1

(*c*) Prove that the necessary and sufficient condition that a given distribution of forces (X, Y, Z) can keep a liquid in equilibrium is that

$$X\left(\frac{\partial Y}{\partial z}-\frac{\partial Z}{\partial y}\right)+Y\left(\frac{\partial Z}{\partial x}-\frac{\partial X}{\partial z}\right)+Z\left(\frac{\partial X}{\partial y}-\frac{\partial Y}{\partial x}\right)=0$$ 7

(*d*) (*i*) Prove that the densities at two points in a fluid at rest under gravity and in the same horizontal plane are equal.

(*ii*) Three liquids, whose densities are in AP, fill a semicircular tube whose bounding diameter is horizontal. Prove that the depth of one of the common surfaces is double that of the other. 3+4=7

Or

A fine circular tube in the vertical plane contains a column of liquid, of density δ, which subtends a right at the centre and a column of density δ' subtending angle α. If θ is the angle that the radius through the common surface makes with the vertical, prove that

$$\tan\theta = \frac{\delta - \delta' + \delta' \cos\alpha}{\delta + \delta' \sin\alpha}$$ 7

5. *(a)* Fill in the blank : 1

The whole pressure of a heavy homogeneous liquid on a plane is equal to the product of the area and the pressure at its ____.

(b) Define force of buoyancy and centre of buoyancy. 2

(c) Find the whole pressure on a triangle, the depths of whose vertices are h_1, h_2, h_3 and the liquid being homogeneous. 2

(d) Prove that the position of the centre of pressure of a plane area is independent of the inclination of the area to the vertical. 6

Or

A circular area of radius a is immersed with its vertical and centre at a depth h. Find the depth of centre of pressure.

(e) A cone floats with its axis horizontal in a liquid of density double its own; find the pressure on its base and prove that if θ be the inclination to the vertical of the resultant thrust on the curved surface and α the semi-vertical angle of the cone, then

$$\tan\theta = \frac{4}{\pi}\tan\alpha$$ 6

Or

A rectangular area is immersed in a heavy homogeneous liquid with two sides horizontal and is divided by horizontal lines into strips on which the total thrusts are equal. If a, b, c are the breadths of three consecutive strips, prove that

$$a(a+b)(b-c) = c(b+c)(a-b)$$

6. (a) State the condition of equilibrium of a body floating in more than one liquid. 2

(b) A thin rod of weight W is loaded at one end with a weight P of insignificant volume. If the rod floats in an inclined position with $\frac{1}{n}$th of its length out of the water, prove that $(n-1)P = W$. 5

(c) Show that the equilibrium is stable, unstable or neutral according as the metacentre is above, below or on the centre of gravity of the body. 5

Or

A solid cone of semi-vertical angle α, specific gravity σ floats in equilibrium in the liquid of specific gravity ρ with its axis vertical and vertex downwards. Determine the condition for which the equilibrium is stable.

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