5 SEM TDC MTH M 4

2017

(November)

MATHEMATICS

(Major)

Course : 504

(Mechanics and Integral Transform)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mechanics)

(a) : Statics

(*Marks* : 25)

1. *(a)* Define the following : 1+1+1=3

(i) Central axis

(ii) Pitch

(iii) Wrench

(b) Find the necessary and sufficient conditions for equilibrium of a rigid body. 7

Or

With usual meaning, show that the quantities $(LX + MY + NZ)$ and $(X^2 + Y^2 + Z^2)$ are invariants for any given system of forces.

2. (a) Write two forces which may be omitted in forming the equation of virtual work. 2

(b) Determine the work done by tension or thrust of a light rod. 6

Or

In a common catenary, prove the following with usual meaning :

(i) $y^2 = c^2 + s^2$

(ii) $x = c\log(\sec\psi + \tan\psi)$

(c) State and prove the principle of virtual work of a system of coplanar forces acting at different points of a rigid body. 7

(b) : Dynamics

(*Marks* : 25)

3. *(a)* Define frequency of a simple harmonic motion. 1

(b) The velocities of a particle along and perpendicular to the radius from a fixed origin are λr and $\mu\theta$. Find the path of the particle and components of acceleration along and perpendicular to the radius vector. 7

Or

Derive the equation of simple harmonic motion.

4. *(a)* Define central force. 1

(b) A particle describes the curve $r^n = a^n \cos n\theta$ under a force F to the pole. Find the law of force. 6

Or

A particle is describing an ellipse under a force to a pole. Find the law of force.

 (*Turn Over*)

5. *(a)* Define product of inertia. 1

(b) Define impressed force. 1

(c) Write the effective forces on a particle along the tangent and normal. 2

(d) State and prove d'Alembert's principle. 6

Or

Find the moment of inertia of a uniform rectangular lamina about a line through its centre and perpendicular to its plane.

GROUP—B

(Integral Transform)

(*Marks* : 30)

6. *(a)* Write the value of the following : 1+1+1=3

(i) $L\{1\}$

(ii) $L\{e^{-t}\}$

(iii) $L\{\cos^2 2t\}$

(b) Find $L\{t^2 e^{3t}\}$. 2

(c) Find $L\{t \sin^2 t\}$. 3

Or

If $f(s) = L\{F(t)\}$, then show that

$$L\{F(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right)$$

7. *(a)* Write the value of $L^{-1}\left\{\frac{1}{s+2}\right\}$. 1

(b) Find : 2+2=4

(i) $L^{-1}\left\{\dfrac{s-2}{s^2-4s+29}\right\}$

(ii) $L^{-1}\left\{\dfrac{1}{(s-6)^2}\right\}$

(c) Evaluate $L^{-1}\left\{\log\dfrac{s+4}{s+2}\right\}$. 3

Or

Evaluate $L^{-1}\left\{\dfrac{1}{s(s+1)^2}\right\}$.

8. (a) Write the value of $L\left\{\dfrac{\partial^2 y}{\partial x^2}\right\}$. 1

(b) Solve the following : 4×2=8

(i) $y''+9y=\cos 2t$, if $y(0)=1$, $y\left(\dfrac{\pi}{2}\right)=-1$

(ii) $y''+y'=t^2+2t$, if $y(0)=4$, $y'(0)=-2$

(c) Find the bounded solution of

$$\frac{\partial y}{\partial x}=2\frac{\partial y}{\partial t}+y,\ y(x,0)=6e^{-3x}$$ 5

Or

Solve :

$$(D^2+2)x - Dy = 1$$

$$Dx + (D^2+2)y = 0;\ D \equiv \frac{d}{dt},$$

with $t > 0$; $x = 0$, $Dx = 0$, $y = 0$, $Dy = 0$, when $t = 0$. Here x and y are functions of t.

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