Total No. of Printed Pages—7

5 SEM TDC MTH M 1

2016

(November)

MATHEMATICS

(Major)

Course : 501

(Logic and Combinatorics, and Analysis—III)

Full Marks : 80

Pass Marks : 32 (Backlog)/24 (2014 onwards)

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) Logic and Combinatorics

(*Marks* : 35)

1. *(a)* State 'True' or 'False' : 1×2=2

(i) '$x-4=6$' is a statement.

(ii) 'What is your name?' is a statement.

(b) *(i)* Write down the converse of $(p \to q)$. 1

(ii) Find the dual of $\sim(p \wedge q) \vee T$. 2

(c) (i) Prove that $p \rightarrow q \equiv \sim p \vee q$. 1

(ii) Prove that the set $\{\rightarrow, \sim\}$ is functionally complete. 4

Or

Using arithmetical representation, prove that $A \vee (A \leftrightarrow A)$ is a tautology. 4

2. (a) Define rules of inferences. 2

(b) Illustrate the derivation

$$A \rightarrow B, \sim (B \vee C) \models \sim A$$ 2

(c) Symbolize the following sentence using predicates : 2

"There are both lawyers and shysters who admire John."

(d) If P_x be 'x is prime', O_x be 'x is odd', D_{xy} be 'x divides y', then translate the following into English : 4

$$(x)(O_x \rightarrow (y)(P_y \rightarrow \sim D_{xy}))$$

Or

Write the formal derivation of the following sentence : 4

"No human beings are quadrupeds.
All women are human beings.
Therefore, no woman is quadruped."

(Continued)

3. *(a)* State the rules of sum and product of counting. 1

(b) In how many ways can we get a total of six while rolling two dice simultaneously? 2

Or

How many solutions does the equation $x_1 + x_2 + x_3 = 11$ have, where x_1, x_2 and x_3 are non-negative integers? 2

(c) State Vandermonde's identity. Prove that

$$\binom{n+1}{r+1} = \sum_{j=r}^{n} \binom{j}{r}$$

where n, r are non-negative integers such that $r \leq n$. 1+3=4

4. *(a)* Define Ramsey number. Show that

$$R(m, n) \leq C(m+n-2, m-1)$$

where m, n are integers greater than 1. 1+3=4

Or

Show that

(i) $R(4, 4) = 18$

(ii) $R(5, 3) = 14$ 2+2=4

(b) How many integers between 1 and 500 are *(i)* divisible by 3 or 5 and *(ii)* divisible by 3 but not by 5 or 6? 4

Or

Find a generating function for a_r = the number of non-negative integral solutions to $e_1 + e_2 + \cdots + e_n = r$, where $0 \le e_i$ for each i. 4

(B) Analysis—III (Complex Analysis)

(*Marks* : 45)

5. *(a)* Write down the conditions for any complex function to be analytic. 1

(b) Derive Cauchy-Riemann equation for a complex function $f(z)$ in Cartesian coordinates. 3

(c) Examine the nature of the function

$$f(z) = \frac{x^2y^5(x+iy)}{x^4+y^{10}};\ z \neq 0,\ f(0) = 0$$

in a region including the origin. 6

Or

Show that the function $f(z) = z^3$ is analytic in a domain D of a complex plane C. 6

6. *(a)* Define rectifiable curve. 1

(b) Show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) = 4\frac{\partial^2}{\partial z \partial \bar{z}}$$ 4

(c) If $u - v = (x-y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of z, find $f(z)$ in terms of z. 5

Or

State and prove Cauchy's theorem. 5

(d) Answer the following (any *one*) : 4

(i) Evaluate

$$\int_C \frac{dz}{z(z-1)}$$

where C is the circle $|z| = 3$.

(ii) Evaluate

$$\int_C \frac{z-1}{(z+1)^2 (z-2)} dz$$

where C is such that $|z - i| = 2$.

7. (a) Define singularities of an analytic function. 2

(b) Expand

$$\frac{1}{z(z^2 - 3z + 2)}$$

for the region $0 < |z| < 1$. 3

(c) Expand e^z in a Taylor's series about $z=0$ and determine the region of convergence. 3

Or

Find Taylor's expansion of $f(z)=\dfrac{z}{z^4+9}$ about $z=0$. 3

8. (a) Find the residues of the function

$$f(z)=\frac{\cot \pi z}{(z-a)^2}$$ 3

(b) Evaluate the following (any *two*) : 5×2=10

(i) $\int_0^{2\pi} e^{-\cos\theta}\cos(n\theta+\sin\theta)\,d\theta$

where n is a positive integer

(ii) $\int_0^{2\pi}\dfrac{d\theta}{1+a^2-2a\cos\theta}$

(iii) $\int_{-\infty}^{\infty}\dfrac{\cos x\,dx}{(x^2+a^2)(x^2+b^2)}$; $a>b>0$

(iv) $\int_{-\infty}^{\infty}\dfrac{x^2}{(x^2+a^2)^3}dx$

where residue is taken to be positive

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