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5 SEM TDC MTH M 3

2016

(November)

MATHEMATICS

(Major)

Course : 503

(Fluid Mechanics)

Full Marks : 80

Pass Marks : 32 (Backlog) / 24 (2014 onwards)

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) Hydrodynamics

(*Marks* : 35)

1. (a) Define ideal fluid. 1

   (b) State whether True or False : 1

   A path line is the curve along which a particular fluid particle travels during its motion.

(c) Find the equation of the streamlines for the flow $\vec{q} = -\hat{i}(3y^2) - \hat{j}(6x)$ at the point (1, 1). 3

(d) Determine the acceleration at the point (2, 1, 3) at $t = 0{\cdot}5$ if $u = yz + t$, $v = xz - t$ and $w = xy$. 4

2. Deduce the equation of continuity in cylindrical coordinates. 6

Or

Show that

$$u = \frac{-2xyz}{(x^2+y^2)^2}, \quad v = \frac{(x^2-y^2)z}{(x^2+y^2)^2} \text{ and } w = \frac{y}{x^2+y^2}$$

are the velocity components of a possible liquid motion. Is this motion irrotational? 6

3. (a) Choose the correct answer : 1

Euler's equation of motion in x direction is

(i) $\frac{Du}{Dt} = X - \frac{1}{\rho}\frac{\partial p}{\partial x}$

(ii) $\frac{Du}{Dt} = X + \frac{1}{\rho}\frac{\partial p}{\partial x}$

(iii) $\frac{\partial u}{\partial t} = X - \frac{1}{\rho}\frac{\partial p}{\partial x}$

(iv) $\frac{\partial u}{\partial t} = X + \frac{1}{\rho}\frac{\partial p}{\partial x}$

(b) If the motion of an ideal fluid, for which density is a function of pressure only, is steady and the external forces are conservative, then prove that there exists a family of surfaces which contain the streamlines and vortex lines. 5

Or

For a steady motion of inviscid incompressible fluid under conservative forces, show that the vorticity $\vec{\omega}$ and velocity $\vec{q}$ satisfies

$$(\vec{q} \cdot \nabla)\vec{\omega} = (\vec{\omega} \cdot \nabla)\vec{q}$$ 5

4. State and prove Kelvin's circulation theorem. 6

Or

A portion of homogeneous fluid is contained between two concentric spheres of radii A and a, and is attracted towards their centre by a force varying inversely as the square of the distance. The inner spherical surface is suddenly annihilated, and when the radii of inner and outer surface of the fluid are r and R, the fluid impinges on a solid ball concentric with these surfaces. Prove that the impulsive pressure at any point of the ball for different values of R and r varies as

$$\left\{(a^2 - r^2 - A^2 + R^2)\left(\frac{1}{r} - \frac{1}{R}\right)\right\}^{\frac{1}{2}}$$ 6

5. (a) Define circulation. 1

(b) Answer *either* (i) *or* [(ii) and (iii)]

(i) Show that if the velocity potential of an irrotational motion is equal to

$$A(x^2 + y^2 + z^2)^{-\frac{3}{2}} \left(z \tan^{-1} \frac{y}{x} \right)$$

the lines of flow lie on the family of surfaces

$$x^2 + y^2 + z^2 = k^{\frac{2}{3}} (x^2 + y^2)^{\frac{2}{3}}$$ 7

Or

(ii) Prove that there cannot be two different forms of irrotational motion for a given confined mass of incompressible inviscid liquid whose boundaries are subject to the given impulses. 3

(iii) If Σ is the solid boundary of a large spherical surface of radius R, containing fluid in motion and also enclosing one or more closed surfaces, then show that the mean value of velocity potential Q on Σ is of the form

$$Q = \left(\frac{M}{R} \right) + C$$

where M, C are constants, provided that the fluid extends to infinity and is at rest there. 4

(B) Hydrostatics

(*Marks* : 45)

6. *(a)* Define specific gravity of a substance. 1

(b) Prove that the densities at two points in a fluid at rest under gravity and in the same horizontal plane are equal. 2

(c) Prove that the surfaces of equal pressure are intersected orthogonally by the lines of force. 3

7. *(a)* A tube in the form of a parabola held with its vertex downwards and axis vertical, is filled with different liquids of densities δ and δ'. If the distance of the free surface of the liquids from the focus be r and r' respectively, show that the distance of their common surface from the focus is

$$\frac{r\delta - r'\delta'}{\delta - \delta'}$$ 6

Or

If the components parallel to the axes of the forces acting on an element of fluid at (x, y, z) be proportional to

$y^2 + 2\lambda yz + z^2$, $z^2 + 2\mu zx + x^2$ and

$$x^2 + 2\nu xy + y^2$$

show that if equilibrium be possible, then $2\lambda = 2\mu = 2\nu = 1$. 6

(b) Prove that the pressure at a depth z below the surface of a homogeneous liquid, at rest under gravity is $p = wz + \Pi$, where Π is the atmospheric pressure and w is the weight of unit volume of the liquid. 5

8. (a) Define centre of pressure. 1

(b) Prove that the whole pressure of a heavy homogeneous liquid on a plane area is equal to the product of the area and the pressure at its centre of gravity. 3

9. (a) Find the centre of pressure of a parallelogram immersed in a homogeneous liquid with one side in the free surface. 6

Or

A triangle ABC is immersed in a liquid, its plane being vertical and the side AB in the surface; if O be the centre of the circumscribed circle of ABC, prove that

$$\frac{\text{Pressure on the } \Delta AOC}{\text{Pressure on the } \Delta OCB} = \frac{\sin 2B}{\sin 2A}$$ 6

(b) A conical glass is filled with water and placed in an inverted position upon a table. Show that the resultant vertical thrust of the water on the glass is two-thirds that on the table. 6

Or

Find the resultant horizontal thrust in an assigned horizontal direction on a curved surface immersed in a heavy homogeneous liquid. 6

10. (a) Fill in the blank : 1

If the ____ coincides with centre of gravity, the equilibrium is neutral.

(b) A body floats partly immersed in one liquid and partly in another. Find the condition of equilibrium. 4

(c) Define stable and unstable equilibrium. 2

11. Prove that the tangent at any point of surface of buoyancy is parallel to the corresponding plane of floatation. 5

Or

A solid body consists of a right cone joined to hemisphere on the same base and floats with the spherical portion partly immersed. Prove that the greatest height of the cone consistent with stability is $\sqrt{3}$ times the radius of the base. 5

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