Total No. of Printed Pages—6

5 SEM TDC MTH M 1

2014

(November)

MATHEMATICS

(Major)

Course : 501

(Logic, Combinatorics and Analysis—III)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) Logic and Combinatorics

(*Marks* : 35)

1. *(a)* What do you mean by validity of an argument? 1

(b) Let p be 'Ravi is rich' and q be 'Ravi is happy'. Write the following statements in symbolic form : 2

(i) Ravi is poor but happy.

(ii) Ravi is neither rich nor unhappy.

(c) Prove that $p \to q = \sim p \vee q$. 3

Or

Using arithmetic representation, prove that $p \vee \sim p$ is a tautology.

(d) Prove that the following argument is valid : 4

$$\begin{array}{c} p \\ p \to q \\ \hline q \end{array}$$

2. (a) Define 'predicate'. 1

(b) What do you mean by consequence? 2

(c) Translate the following in symbols using quantifiers, predicate letter or individual variable : 3

(i) All judges are lawyers.

(ii) Not all lawyers are judges.

(iii) Any one can do this.

(d) Answer any *one* of the following : 4

(i) Let $S(x)$: x is a soldier.

$G(x)$: x is ignorant.

$I(x)$: x is Indian.

Then show that $(\exists x)(S(x) \wedge G(x))$, $(\exists x)(I(x) \wedge G(x)) \models (\exists x)(S(x) \wedge I(x))$

(ii) Every member of the committee is old and academician. Some members of the committee are rich. Therefore, there are some rich academicians. Derive it mathematically.

3. *(a)* State Vander Monde's identity. 1

(b) How many solutions are thereof $x+y+z=17$ such that $x \geq 1$, $y \geq 2$ and $z \geq 3$? 2

(c) Define Stirling numbers. Prove that

$$[x]^n = \sum_{k=1}^{n} U(n, k)[x]_k$$

where $u(n, k) = \frac{\lfloor n}{\lfloor k} C(n-1, k-1)$ 1+3=4

Or

Define multinomial theorem. Prove that in $(2x-3y+5z)^8$, the coefficient of $x^3y^3z^2$ is $(2)^3(-3)^3(5)^2(560)$. 4

4. *(a)* State the principle of inclusion-exclusion for three sets. 1

(b) How many integers between 500 to 1000 are divisible by 3 or 5? 3

(c) There are 300 boxes with mangoes. Each contains no more than x mangoes. Find the maximum possible value of x such that 3 boxes containing equal number of mangoes. 4

Or

Define exponential-generating function. Find the generating function for the sequence 1, 3, 9, ..., 3^n, ...

(B) Analysis—III (Complex Analysis)

(*Marks* : 45)

5. (a) Define an analytic function. 1

(b) Show that $e^x(\cos y + i\sin y)$ is an analytic function. Find its derivative. 2+2=4

(c) Show that the function $f(z) = \sqrt{|xy|}$ satisfies Cauchy-Riemann equation at the origin but not analytic at that point. 5

Or

Find the imaginary part of the analytic function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2$.

6. *(a)* Define simply connected domain. 1

(b) Find the value of the integral

$$\int_0^{1+i} (x - y + ix^2)\, dz$$

along the straight line from $z = 0$ to $z = 1 + i$. 2

(c) State and prove Cauchy's integral formula. 5

(d) Evaluate (any *two*) : 6

(i) $\int_C \frac{e^z}{(z-1)(z-4)} dz$, where C is the circle $|z| = 2$ by using Cauchy's integral formula

(ii) $\int_C \frac{3z^2 + z}{z^2 - 1} dz$, where C is $|z - 1| = 1$

(iii) $\int_C \frac{e^{zt}}{z^2 + 1} dz$, where $t > 0$, C is $|z| = 3$

7. *(a)* State and prove Laurent's series. 1+5=6

(b) Expand $\frac{1}{z^2 + 5z + 6}$ in $|z| < 1$. 2

8. *(a)* Define removable singularity of a function $f(z)$. 1

(b) Find the pole of $f(z) = \frac{\sin(z-a)}{(z-a)^4}$. 2

(Turn Over)

(c) Evaluate (any *two*) : 5×2=10

(i) $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$

(ii) $\int_0^{2\pi} \frac{d\theta}{5-4\sin\theta}$

(iii) $\int_C \frac{dz}{z^2(z+1)(z-1)}$, where C is $|z|=3$

(iv) $\int_{-\infty}^{\infty} \frac{\cos ax}{x^2+1} dx$

★★★