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5 SEM TDC MTH M 4

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(November)

MATHEMATICS

(Major)

Course : 504

(Mechanics and Integral Transforms)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(MECHANICS)

(a) : Statics

(*Marks* : 25)

1. *(a)* Write the value of the pitch of the wrench $(\vec{R}, \vec{G})$. 1

(b) Define screw. 2

(c) Prove that a system of forces can be reduced to a single force acting through an arbitrary chosen point and a couple whose axis passes through that point. 7

Or

Find the equation of null plane of a given point (a, b, c) referred to coordinate system *oxyz*.

2. *(a)* Define virtual work. 1

(b) Write the name of one force which can be omitted in forming the equation of virtual work. 1

(c) Establish the relation between x and s for a common catenary. 2

(d) State and prove the principle of virtual work for a system of coplanar forces acting at different points of a rigid body. 6

(e) Derive the intrinsic equation of common catenary. 5

Or

A regular hexagon *ABCDEF* consists of six equal uniform rods, each of weight w, freely jointed together. The hexagon rests in a vertical plane and *AB* is in contact with a horizontal table. If *C* and *F* be connected by a light string, then find the tension of the string.

(b) : Dynamics

(*Marks* : 25)

3. *(a)* Define radial velocity of a particle. 1

(b) Define the amplitude of a simple harmonic motion. 1

(c) Find the radial and transverse velocity components of a particle. 6

Or

A particle describes the curve $r = ae^{m\theta}$ with a constant velocity. Find the components of velocity along radius vector and perpendicular to it.

4. (a) Write the name of the orbit of a particle moving under a central force. 1

(b) If a particle moves upward in a resisting medium, then write the direction along which the resisting force acts. 1

(c) A particle describes the curve $p^2 = ar$ under a force F to the pole. Find the law of the force. 5

Or

A particle falls under gravity from rest in a medium whose resistance varies as the velocity. Find the relation between x and t.

5. (a) Define effective force on a particle. 1

(b) Let (x, y, z) be the coordinates of a point mass m. Then write the moment of inertia of the point mass with respect to the origin. 1

(c) Prove the theorem of perpendicular axes of moment of inertia. 3

(d) Find the moment of inertia of a plane lamina of length $2a$ and breadth $2b$ about a line through its centre and parallel to x-axis. 5

Or

Deduce the general equation of motion of a rigid body from D'Alembert's principle.

GROUP—B

(INTEGRAL TRANSFORMS)

(*Marks* : 30)

6. (a) Write the value of $L\{t\}$. 1

(b) Find $L\{\sin 4t\}$. 2

(c) Evaluate $L\{\cosh t\}$. 2

(d) Evaluate (any *one*) : 3

(i) $L\{\sin^3 t\}$

(ii) $L\{e^t \cos t\}$

7. (a) Write the value of $L^{-1}\left\{\frac{1}{s+1}\right\}$. 1

(*Turn Over*)

(b) Evaluate : 2+2

(i) $L^{-1}\left\{\dfrac{s-2}{s^2-4s+20}\right\}$

(ii) $L^{-1}\left\{\dfrac{1}{(s+1)(s-2)}\right\}$

(c) Evaluate $L^{-1}\left\{\dfrac{s}{s^2+1}\right\}$. 3

Or

Evaluate $L^{-1}\left\{\log\left(1+\dfrac{1}{s^2}\right)\right\}$.

8. *(a)* If $L\{y(x, t)\} = \overline{y}(x, s)$, then write the value of $L\left\{\dfrac{\partial y}{\partial t}\right\}$. 1

(b) Solve $\dfrac{d^2y}{dt^2} + y = 0$, using Laplace transform, with conditions $y(0) = 1$, $y'(0) = 0$. 3

(c) Solve

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 6y = 2$$

using Laplace transform with conditions $y(0) = 1$, $y'(0) = 0$. 5

Or

Solve $\frac{d^2y}{dt^2} + y = t;$ using Laplace transform with conditions $y'(0) = 1$, $y(\pi) = 0$.

(d) Solve

$$\frac{d^2y}{dt^2} + t\frac{dy}{dt} - y = 0$$

using Laplace transform with conditions $y(0) = 0,\ y'(0) = 1.$ 5

Or

Solve $\frac{\partial^2 y}{\partial x^2} - \frac{\partial^2 y}{\partial t^2} = xt;$ when $t = 0,\ y = 0,$ $\frac{\partial y}{\partial t} = 0,$ and $y(0,\ t) = 0.$

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