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5 SEM TDC MTH M 1

2013

(November)

MATHEMATICS

(Major)

Course : 501

(Logic and Combinatorics, and Analysis—III)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) Logic and Combinatorics

(*Marks* : 35)

1. *(a)* Define truth function. 1

(b) Let P be 'it is cold' and Q be 'it is raining'. Give verbal sentence which describes each of the following : 2

(i) $P \vee \sim Q$

(ii) $\sim P \wedge \sim Q$

(c) Construct the truth table for $(p \wedge q) \rightarrow p$. State whether it is a tautology or not. 3

(d) Prove that every truth function can be generated by ~, $\wedge$ and $\vee$. Can you generate a truth function by using ~ and $\wedge$ only? 4

Or

Give the arithmetic representation of the form $\sim P$, $P \vee Q$, $P \wedge Q$, $P \rightarrow Q$. Also show that $P \vee \sim P = 1$.

2. (a) What do you mean by equivalent statements? 1

(b) Write the rule p and rule t. 2

(c) Translate into symbols : 3

(i) Not all birds can fly.

(ii) Anyone can do it.

(iii) Some people are intelligent.

(d) Derive any *one* of the following : 4

(i) Everyone who buys a ticket receives a prize. Therefore, if there is no prize, there nobody buys ticket.

(ii) All men are mortal. Ram is man. Hence Ram is mortal.

3. *(a)* State the Pascal's identity. 1

(b) Find the coefficient of $x_1^2 x_2^3 x_4^5 x_5^7$ in $(x_1 + x_2 + x_3 + x_4 + x_5)^{17}$. 2

(c) Define Ramsey number $R(p, q)$. Prove that $R(4, 3) = 9$. 4

Or

Define Catalan numbers. Prove that nth Catalan number

$$C_n = \frac{2^{n-1}\{1 \cdot 3 \cdot 5 \cdot \cdots (2n-3)\}}{\lfloor n}$$

4. *(a)* State the pigeonhole principle. 1

(b) How many integers between 100 and 700 are divisible by 3 or 5? 3

(c) Prove that given any 12 natural numbers we can choose 2 of them such that their difference is divisible by 11. 4

Or

Define binomial generating function. Find both binomial and exponential generating functions for the sequence 2, 2, 2, 2, ⋯.

(B) Analysis—III (Complex Analysis)

(*Marks* : 45)

5. *(a)* State the condition under which a function is said to be analytic. 1

(b) Define harmonic function. Show that $u(x, y) = x^4 - 6x^2y^2 + y^4$ is harmonic. 3

(c) State and prove the necessary conditions for a function $f(z) = u + iv$ to be analytic at all points in a region R. 6

Or

Show that

$$f(z) = \frac{x^2y^5(x+iy)}{x^4 + y^{10}}, \quad z \neq 0$$

$$f(0) = 0 \quad , \quad z = 0$$

is not analytic at the origin, although Cauchy-Riemann equations are satisfied. What is your opinion in this case?

6. *(a)* Define Jordan's arc. 1

(b) Find the value of the integral

$$\int_0^{1+i} (x^2 - iy)\, dz$$

where $y = x$. 2

(c) State and prove Cauchy's integral theorem. 5

(*Continued*)

(d) If a function $f(z)$ is analytic for all finite values of z and is bounded, then show that it is constant. 6

Or

Evaluate :

(i) $\int_C \frac{2z+1}{z^2+z} dz$, where C is $|z| = \frac{1}{2}$

(ii) $\int_C \frac{dz}{z-a}$, where C is $|z-a| = r$

7. (a) State and prove Taylor's series. 1+5=6

(b) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent's series, where $|z| > 3$. 2

8. (a) Define an isolated singular point of a function $f(z)$. 1

(b) Discuss the singularity of

$$\frac{\cot \pi z}{(z-a)^2}$$

at $z = a$, $z = \infty$. 2

(c) Evaluate (any *two*) : 5×2=10

(i) $\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$

(ii) $\int_0^{\infty} \frac{dx}{1+x^2}$

(iii) $\int_C \frac{z^2 dz}{(z-1)^2(z+2)}$, where C is $|z|=3$

(iv) $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$

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