5 SEM TDC PHY M 1

2019

(November)

PHYSICS

(Major)

Course : 501

(Mathematical Physics)

Full Marks : 60

Pass Marks : 24/18

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following (any *six*) : 1×6=6

(a) $\frac{1}{D^2-1}x^2$ is equal to

(i) x^2+2x+2

(ii) $-(x^2+2x+2)$

(iii) $2x-x^2$

(iv) $-(2x-x^2)$

(b) The equation $y^2 dx + (x^2 - xy - y^2) dy = 0$ is

(i) exact

(ii) inexact

(iii) Both (i) and (ii)

(iv) None of the above

(c) If $P_n(x)$ be a Legendre polynomial of order n, then $\left[\frac{d}{dx} P_n(x)\right]_{x=1}$ is

(i) $n(n+1)$

(ii) $\frac{n(n+1)}{2}$

(iii) $\frac{1}{2}(n+1)$

(iv) $\frac{n+1}{2n}$

(d) Which of the following functions is not analytic everywhere?

(i) $e^{\sin z}$

(ii) $\log z$

(iii) e^z

(iv) $\sin z$

(e) Residues at two poles of the function $f(z) = \frac{z}{z^2 + 4}$ are

(i) equal and opposite in sign

(ii) equal with same sign

(iii) all together different

(iv) inverse of each other

(f) According to Fourier expansion of x^2, the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is

(i) $\frac{\pi^2}{3}$

(ii) $\frac{\pi^2}{4}$

(iii) $\frac{\pi^2}{6}$

(iv) $\frac{\pi^2}{12}$

(g) The Fourier series expansion of x^2 in the interval $-\pi < x < \pi$ contains

(i) sine and cosine terms of integral multiples of x

(ii) only cosine terms of integral multiple of x

(iii) only sine terms of integral multiple of x

(iv) only cosine terms of even multiples of x

2. Answer any *six* of the following : 2×6=12

(a) Find the Fourier series expansion for the output of a half-wave rectifier having period $\frac{2\pi}{\omega}$, where ω is angular frequency.

(b) Show that

$$(n+1)\, P_{n+1}(x) = (2n+1)\, x\, P_n(x) - n\, P_{n-1}(x)$$

(c) State Cauchy's theorem and Cauchy's integral formula.

(d) Show that

$$\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right) = \frac{2\pi}{\sqrt{3}}$$

(e) Find the Fourier series of the function $f(x) = e^x$ in the interval $-\pi < x < \pi$.

(f) Examine whether $\sin z$ is an analytic function of z.

(g) What do you mean by order and degree of a differential equation? Explain with two examples.

3. (a) For the Legendre polynomials, prove the following integral : 5

$$\int_{-1}^{+1} P_l(x)\, P_m(x)\, dx = 0 \quad (l \neq m)$$

(b) What are differential equations? Explain the importance of differential equations in physical problems. What do you mean by the term solution of differential equation? 1+1+1=3

Or

Show that for $|x|$ is large, $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$. 3

(c) Solve the following differential equation by Frobenius method : 5

$$2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1 - x^2)y = 0$$

Or

Solve the equation in series

$$9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$$

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(d) Show that

$$\beta(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)} \quad \text{for } m > 0,\ n > 0$$

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(e) Find the solution of the differential equation

$$(x+2y)(dy - 2dx) + dx + 2dy = 0$$ 4

Or

Solve :

$$y\,(y^2 - 2x^2)\,dx + x(2y^2 - x^2)\,dy = 0$$ 4

4. (a) Prove Cauchy's integral formula and apply it to evaluate the integral

$$\int_C \frac{2z+1}{z^2+z}\,dz, \text{ where } C \text{ is } |x| = \frac{1}{2}$$ 3+2=5

(b) Prove by contour integration method

$$\int_0^{\pi} \frac{a\,d\theta}{a^2 + \sin^2\theta} = \frac{\pi}{\sqrt{1+a^2}},\ a > 0$$ 4

Or

Obtain Laurent series expansion of

$$f(z) = \frac{1}{z^2 - 3z + 2} \text{ in the region } 1 < |z| < 2$$

5. (a) By using Fourier series, expand the function $y = \cos 2x$ in a series of sines in the interval $(0, \pi)$. 4

(b) Find the series of sines and cosines of multiples of x which represents $f(x)$ in the interval $-\pi < x < \pi$, where

$$f(x) = \begin{cases} 0 & , \text{ when } -\pi < x \le 0 \\ \dfrac{\pi x}{4} & , \text{ when } 0 < x \le \pi \end{cases}$$

and hence deduce

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\cdots$$ 5

Or

What are odd and even functions? Find a series of sines and cosines of multiple of x; which will represent $x + x^2$ in the interval $-\pi < x < \pi$. Deduce that

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\cdots$$ 5

(c) A sawtooth wave is given by

$$f(x) = \begin{Bmatrix} 0, & \text{for } -\pi < x < 0 \\ h, & \text{for } 0 \le x < \pi \end{Bmatrix}$$

Show that $f(x) = \frac{h}{2} + \frac{2h}{\pi} \sum_{n=1}^{\infty} \frac{\sin nx}{n}$ 3

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