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5 SEM TDC PHY M 1

2016

(November)

PHYSICS

(Major)

Course : 501

(Mathematical Physics)

Full Marks : 60

Pass Marks : 24 (Backlog)/18 (2014 onwards)

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option : 1×6=6

(a) Which of the following integrals is non-vanishing?

(i) $\int_{-1}^{+1} xP_n \frac{dP_m}{dx} dx$ for $n > m$

(ii) $\int_{-1}^{+1} P_n(x) dx$

(iii) $\int_{-1}^{+1} x^2 P_5(x) dx$

(iv) $\int_{-1}^{+1} P_0(x) dx$

(b) Given $\Gamma(3)\Gamma\left(\frac{5}{2}\right) = A\Gamma(6)$, find A.

(i) $\sqrt{\pi}$

(ii) $\sqrt{\pi}/2$

(iii) $\sqrt{\pi}/2^3$

(iv) $\sqrt{\pi}/2^5$

(c) If $u = x^3 - 3xy^2$, the analytic function $f(z) = u + iv$ will be

(i) z^3

(ii) z^{-3}

(iii) $|z|^3$

(iv) None of the above

(d) What is the ratio of coefficients of z^n and $\frac{1}{z^n}$ in the Laurent's expansion of the function $\cosh\left(z + \frac{1}{z}\right)$?

(i) 0

(ii) $\frac{1}{2}$

(iii) 1

(iv) None of the above

(e) The value of a_0 in the Fourier series of t^2 in the interval $-\pi < t < \pi$ is

(i) 0

(ii) $\frac{\pi^2}{3}$

(iii) $\frac{\pi^2}{8}$

(iv) $\frac{\pi^2}{4}$

(f) Using Fourier integral, the value of $\int_0^{\infty} \frac{\cos xu}{1+u^2} du \; (x > 0)$ is found to be

(i) $\frac{\pi}{2}$

(ii) $\frac{\pi}{2} e^x$

(iii) $\frac{2}{\pi} e^{-x}$

(iv) $\frac{\pi}{2} e^{-x}$

2. (a) Prove that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$ 2

(b) Find the value of a_n in the Fourier series of $f(x)$ in the interval $(-\pi, \pi)$, where

$$f(x) = \pi + x, \text{ when } -\pi < x < 0$$
$$= \pi - x, \text{ when } 0 < x < \pi$$ 2

(c) Prove that $P_{2m}(-\mu) = P_{2m}(\mu)$. 2

(d) Express the integral $I = \int_0^\infty \frac{x^3}{(1+x)^5} dx$ in terms of beta and gamma functions and hence find its value. 2

(e) Using Cauchy's integral formula, evaluate the integral $\oint \frac{z^2}{(z^2-1)} dz$ around the unit circle with centre at $z = 1$. 2

(f) If $u(x, y) = x^2 - y^2$ is the real part of an analytic function $f(z) = u + iv$, find v. 2

3. (a) Solve the equation $y'' - y = 0$ with $y(0) = 4$, $y'(0) = -2$. 3

(b) Find the solution of the non-homogeneous equation $y'' + 4y = 8x^2$. 3

(c) Prove that

$$P_n(x) = \frac{1}{2^n n!}\left(\frac{d}{dx}\right)^n (x^2 - 1)^n$$ 4

(d) Prove that

$$(2n+1)\,xP_n(x) = (n+1)\,P_{n+1}(x) + nP_{n-1}(x)$$ 4

Or

Prove that Legendre polynomial $P_n(\mu)$ is the coefficient of h^2 in $(1-2\mu h+h^2)^{-1/2}$. 4

4. (a) Prove that if $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then u and v satisfy $\nabla^2 u = 0$ and $\nabla^2 v = 0$. 4

(b) Prove that if $f(z)$ is an analytic function on and within the closed contour c, the value of $f(z)$ at a point $z = \varepsilon$ inside c is given by

$$f(\varepsilon) = \frac{1}{2\pi i}\oint \frac{f(z)}{z-\varepsilon}\,dz$$ 4

(c) Answer any *two* from the following : 3×2=6

(i) Show that the triangle whose vertices are the points z_1, z_2, z_3 in Argand diagram will be equilateral if

$$z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$$

(ii) If $f(z)$ is an analytic function of $|z|$, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\,|f'(z)|^2$$

(iii) Obtain the expansion

$$f(z) = f(a) + 2\left\{\frac{z-a}{2} f'\left(\frac{z+a}{2}\right) + \frac{(z-a)^3}{2^3\,3!} f'''\left(\frac{z+a}{2}\right) + \frac{(z-a)^5}{2^5\,5!} f^{(5)}\left(\frac{z+a}{2}\right) + \cdots\right\}$$

and determine its range of validity.

5. *(a)* Find an even function of x which is equal to kx for $0 \le x \le l/2$ and is $k(l-x)$ for $l/2 \le x \le l$ 3

(b) Find the series of sines and cosines of multiples of x which represents $f(x)$ in the interval $-\pi < x < \pi$, where

$$f(x) = 0, \quad \text{when } -\pi < x < 0$$
$$= \frac{\pi x}{4}, \quad \text{when } 0 < x < \pi$$

4

(c) Show that the rectified current through a half-wave rectifier is

$$I(t) = \frac{I_0}{\pi} - \frac{2I_0}{\pi}\left(\frac{1}{3}\cos 2\omega t + \frac{1}{15}\cos 4\omega t + \cdots\right) + \frac{1}{2} I_0 \sin \omega t$$

4

(d) State and prove Parseval's theorem. 3

Or

Obtain the Fourier series for a triangular wave given by

$$y = 0 \quad \text{at} \quad t = 0$$
$$y = a \quad \text{at} \quad t = T/2$$
$$y = 0 \quad \text{at} \quad t = T$$

3

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