5 SEM TDC PHY M 1

2013

(November)

PHYSICS

(Major)

Course : 501

(Mathematical Physics)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option : 1×6=6

(a) In the general solution of second-order differential equation

$$\frac{d^2y}{dx^2} - 2\alpha\frac{dy}{dx} + \alpha^2 y = 0$$

one term contains $e^{\alpha x}$. Then its second term will be a constant times

(i) $e^{-\alpha x}$

(ii) $xe^{-\alpha x}$

(iii) $xe^{\alpha x}$

(iv) $\frac{1}{x}e^{\alpha x}$

(b) The value of the integral

$$\int_{-1}^{+1} xP_n(x)P_{n-1}(x)\,dx$$

is

(i) 0

(ii) $\frac{2}{(2n+1)}$

(iii) $\frac{1}{(4n^2-1)}$

(iv) $\frac{2n}{(4n^2-1)}$

(c) The value of

$$\int_0^\infty \sqrt{\frac{\lambda}{y}}\, e^{-\lambda y}\, dy$$

is

(i) $\Gamma\left(\frac{1}{2}\right)$

(ii) $\Gamma\left(\frac{3}{2}\right)$

(iii) $\frac{\sqrt{\pi}}{2}$

(iv) $\Gamma\left(\frac{\lambda}{2}\right)$

(d) The value of

$$\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta} d\theta$$

is

(i) π

(ii) $\frac{\pi}{3}$

(iii) $\frac{\pi}{6}$

(iv) $\frac{2\pi}{3}$

(e) Residue of the function

$$f(z) = \frac{z^2}{z^2 + 4}$$

at $z = 2i$ is

(i) $e^{i\pi/2}$

(ii) $e^{i\pi}$

(iii) $e^{3i\pi/2}$

(iv) None of the above

(f) Expanding the following output from a half-wave rectifier

$$v(t) = \begin{cases} 0 & \text{for } -\frac{T}{2} < t < 0 \\ v_0 \sin \omega t & \text{for } 0 < t < \frac{T}{2} \end{cases}$$

with $v(t+T) = v(t)$, the value of

$$\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \cdots$$

is

(i) $\frac{\pi}{4}$

(ii) 1

(iii) $\frac{\pi}{2}$

(iv) $\frac{1}{2}$

2. Answer any *three* from the following : 2×3=6

(a) Solve :

$$\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$$

(b) Find a solution of the non-homogeneous equation

$$\frac{d^2y}{dx^2} + 4y = 8x^2$$

(c) Show that

$$P_n(-x) = (-1)^n P_n(x)$$

(d) Prove that

$$\beta(m, n) = \beta(n, m)$$

3. (a) If $f(z) = u + iv$ is an analytic function and $\vec{P} = v\hat{i} + u\hat{j}$ is a vector, then show that div $\vec{F} = 0$ and curl $\vec{F} = 0$ are equivalent to Cauchy-Riemann equation. 3

(b) Find the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ using Fourier series. 3

4. (a) Find the solutions of the equation

$$\frac{d^2y}{dx^2} + \omega^2 y = 0$$

using Frobenius's method. 6

(b) Show that

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}} dx = \frac{\Gamma(m)\,\Gamma(n)}{a^n (a+1)^m \Gamma(m+n)}$$ 4

Or

Show that

$$\int_0^1 \frac{x^n dx}{\sqrt{1-x^2}} = \frac{1 \cdot 3 \cdot 5 \cdot \cdots (n-1)}{2 \cdot 4 \cdot 6 \cdot \cdots n} \cdot \frac{\pi}{2}$$

if n is even integer.

(c) Prove that

$$\int_{-1}^{+1} P_m(x) P_n(x)\, dx = \frac{2}{2n+1}\delta_{mn}$$ 6

(d) Consider the equation

$$L\frac{di}{dt} + \frac{q}{c} = 0$$

where $q = \int i\, dt$, $L =$ coefficient of self-induction and $C =$ capacitance. Solve the above equation and determine the constants in such a way that I is the maximum current and $i = 0$, when $t = 0$. 4

5. Answer any *two* questions from the following : 5×2=10

(a) Prove that if $f(z)$ is an analytic function of z and $f'(z)$ is continuous at each point within and on a closed contour C, then

$$\int_C f(z)\, dz = 0$$

(b) Obtain the Laurent series expansion of

$$f(z) = \frac{1}{z^2 - 3z + 2}$$

in the region $1 < |z| < 2$.

(c) Prove that

$$I = \int_0^{2\pi} \frac{d\theta}{2+\cos\theta} = \frac{2\pi}{\sqrt{3}}$$

6. (a) Find the series of sines and cosines of multiples of x which represents $f(x)$ in the interval $-\pi < x < \pi$, where

$$f(x) = 0 \quad \text{when } -\pi < x < 0$$
$$= \frac{\pi x}{4} \quad \text{when } 0 < x < \pi$$

and hence deduce that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$ 6

(b) Develop $f(x)$ in Fourier series in the interval $(-2, 2)$ if $f(x) = 0$ for $-2 < x < 0$ and $f(x) = 1$ for $0 < x < 2$. 6

Or

If

$$f(x) = \begin{cases} x & \text{for } 0 \le x \le \frac{\pi}{2} \\ \pi - x & \text{for } \frac{\pi}{2} \le x \le \pi \end{cases}$$

express it by a sine series and also by a cosine series.

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