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3 SEM TDC MTH M 2

2019

(November)

MATHEMATICS

(Major)

Course : 302

(Coordinate Geometry and Algebra—I)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Coordinate Geometry)

SECTION—I

(2-Dimension)

1. *(a)* State True or False : 1

The coordinates of a point are invariant under change of axes.

(b) Find the transformed equation of the line $x\cos\alpha + y\sin\alpha = p$, when the axes are rotated through an angle α. 2

(c) Transform the equation

$$3(12x-5y+39)+2(5x+12y-26)^2=169$$

taking the lines $12x-5y+39=0$ and $5x+12y-26=0$ as the new axes of x and y respectively. 2

2. (a) Write down the condition that a general equation of second degree represents a pair of straight lines. 1

(b) Prove that the product of the perpendiculars from the point (x', y') on the lines $ax^2+2hxy+by^2=0$ is

$$\frac{ax'^2+2hx'y'+by'^2}{\sqrt{(a-b)^2+4h^2}}$$ 3

Or

Show that the angle between one of the lines $ax^2+2hxy+by^2=0$ and one of the lines $(a-\lambda)x^2+2hxy+(b-\lambda)y^2=0$ is equal to the angle between the other two lines of the system.

(c) Prove that the equation

$$x^2+6xy+9y^2+4x+12y=0$$

represents a pair of parallel straight lines and find the distance between them. 3

(d) Show that the equation

$$y^3 - x^3 + 3xy(y - x) = 0$$

represents three lines equally inclined to one another. 5

Or

Prove that the two pair of lines

$$ax^2 + 2hxy + by^2 = 0 \text{ and}$$
$$a^2x^2 + 2h(a+b)xy + b^2y^2 = 0$$

have the same bisectors.

3. (a) Define a non-central conic. 1

(b) Find the condition that the line $lx + my + n = 0$ is a tangent to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$ 3

(c) Reduce the equation

$$9x^2 + 24xy + 16y^2 - 126x + 82y - 59 = 0$$

to the standard form. 6

Or

Define pole and polar. Find the equation of the polar of the point (2, 3) with respect to the conic

$$x^2 + 3xy + 4y^2 - 5x + 3 = 0$$

SECTION—II

(3-Dimension)

4. (a) If $\cos\alpha$, $\cos\beta$, $\cos\gamma$ be the direction cosines of a line, then prove that

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$ 1

(b) Find the direction cosines of a line that makes equal angles with the coordinate axes. 2

(c) The projection of a line on the axes are 6, 2, 3. Find the length of the line. 3

Or

Find the equation of the plane passing through the points (1, 1, 2) and (2, 4, 3) and perpendicular to the plane $x - 3y + 7z + 5 = 0$.

(d) A plane cuts the coordinate axes at A, B, C such that the centroid of the ΔABC is the point (ξ, η, ζ). Show that the equation of the plane is

$$\frac{x}{\xi} + \frac{y}{\eta} + \frac{z}{\zeta} = 3$$ 4

Or

A variable plane is always at a constant distance p from the origin and meets the axes at A, B, C. Prove that the locus of the point of intersection of the three planes through A, B, C parallel to the coordinate planes is given by

$$x^{-2} + y^{-2} + z^{-2} = p^{-2}$$

5. (a) Find the shortest distance between the lines

$$\frac{x+a}{12} = \frac{y}{6} = \frac{z}{-1}; \quad \frac{x}{6} = \frac{y+2a}{6} = \frac{z-a}{1}$$ 3

(b) Find the length of the shortest distance between the lines

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}; \quad \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$ 5

Or

Find the shortest distance between the axis of z and the line

$$ax + by + cz + d = 0; \quad a'x + b'y + c'z + d' = 0$$

GROUP—B

(Algebra—I)

6. (a) Does the set of all odd integers form a group with respect to addition? 1

(b) Give an example of an Abelian group. 1

(c) Show that the identity element in a group is its own inverse. 2

(d) Answer any *two* questions : 3×2=6

(i) If a, b are any two elements of a group G, then show that the equations $ax = b$ and $ya = b$ have unique solutions in G.

(ii) Show that if every element of a group G is its own inverse, then G is Abelian.

(iii) Define cyclic groups. Show that every cyclic group is Abelian.

7. Answer any *two* questions : 5×2=10

(a) Let H be a subgroup of a finite group G. Then show that the order of H is a divisor of the order of G.

(b) Let H, K are two subgroups of a group G, then prove that HK is a subgroup of G, iff $HK = KH$.

(c) If H and K are normal subgroups of a group G, then prove that $H \cap K$ is a normal subgroup of G. Write the condition so that $H \cup K$ is also a normal subgroup of G.

8. (a) Define permutation group. 1

(b) Let H and K be finite subgroups of a group G. Then prove that

$$O(HK) = \frac{O(H)\,O(K)}{O(H \cap K)}$$ 4

Or

State and prove Cayley's theorem.

9. Answer any *two* questions : 5×2=10

(a) Define homomorphism of groups. Let $f : G \rightarrow G'$ be a homomorphic mapping of a group G into a group G'. Then show that $f(a^{-1}) = [f(a)]^{-1}$; where $a \in G$ and $f(a) \in G'$.

(Turn Over)

(b) Prove that any two infinite cyclic groups are isomorphic to each other.

(c) Define Kernel of a group homomorphism. Prove that the Kernel K of a homomorphism ϕ of a group G into group G' is a normal subgroup of G.

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