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(November)

MATHEMATICS

(Major)

Course : 301

[Analysis—I (Real Analysis)]

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Differential Calculus)

(*Marks* : 35)

1. (a) If $y = \log(a+x)$, find y_n. 1

(b) If $y = e^{3x} \sin 4x$, find y_n. 2

(c) Evaluate

$$\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$$ 3

Or

Find $\frac{ds}{dx}$ for the curve $y^2 = 4ax$.

(d) If $y = (\sin^{-1} x)^2$, then show that

$$(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} - n^2 y_n = 0$$ 4

Or

Find the radius of curvature at any point (x, y) for the curve $y = \log \sin x$.

2. (a) Write the geometrical interpretation of the Lagrange's mean value theorem. 2

(b) State and prove Rolle's theorem. 4

(c) Expand e^x in a finite series in powers of x with Lagrange form of remainder. 4

Or

If $f'(x) = 0$ for all values of x in an interval, then show that $f(x)$ is constant in that interval.

3. (a) Find $\frac{\partial f}{\partial x}$, where $f = e^{x^2 + xy}$. 1

(b) If

$$u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$$

then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$. 4

Or

If

$$u = \sin^{-1}\frac{x^2 + y^2}{x + y}$$

then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

4. (a) Define Jacobian of a function of two variables. 1

(b) Write the necessary condition for a function $f(x, y)$ to have an extreme value at (a, b). 1

(c) If

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

then show that the partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ exist at (0, 0). 3

(d) Find the maximum and minimum value of the function

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20$$ 5

Or

If $v = v(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$, then find $\frac{\partial^2 v}{\partial x^2}$.

GROUP—B

(Integral Calculus)

(*Marks* : 20)

5. (a) Write the condition, when $\int_0^{2a} f(x)\,dx = 0$. 1

(b) Show that

$$\int_0^{\pi/2} \log\sin x\,dx = \int_0^{\pi/2} \log\cos x\,dx$$ 2

(c) Evaluate any *one* : 3

(i) $\int_0^{\pi/2} \log\tan x\,dx$

(ii) $\int_0^1 \frac{x^6}{\sqrt{1-x^2}}\,dx$

(d) Obtain the reduction formula for

$$\int_0^{\pi/2} \sin^n x\,dx$$ 4

Or

Evaluate

$$\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$$

6\. (a) Write the area of the surface of the solid obtained on revolving about x-axis, the arc of the curve $y = f(x)$ intercepted between the points whose abscissas are a and b. 2

(b) Find the length of the arc of the parabola $y^2 = 4ax$ cut off by its latus rectum. 4

(c) Find the volume of the solid obtained by revolving one arc of the cycloid

$$x = a(\theta + \sin\theta),\ y = a(1 + \cos\theta)$$ 4

Or

Find the surface generated by the revolution of an arc of the catenary

$$y = c\cosh\frac{x}{c}$$

about x-axis.

GROUP—C

(Riemann Integral)

(*Marks* : 25)

7. (*a*) Write the condition when the function f is Riemann integrable over $[a, b]$. 1

(*b*) Choose the correct answer for the following : 1

If $\int_a^b f(x)\,dx$ exists, then

(*i*) f is bounded

(*ii*) f is unbounded

(*iii*) interval $[a, b]$ is finite

(*iv*) Both (*i*) and (*iii*)

(*c*) Show that x^2 is integrable on any interval $[0, a]$. 3

(*d*) Prove that every continuous function is integrable. 3

Or

If P_1 is a refinement of a partition P, then for a bounded function f, show that $U(P_1, f) \le U(P, f)$.

8. (a) Prove that if a function f is continuous on $[a, b]$, then there exists a number c in $[a, b]$, such that

$$\int_a^b f\,dx = f(c)(b-a)$$ 4

(b) If f is continuous and positive on $[a, b]$, then show that $\int_a^b f\,dx$ is also positive. 3

Or

Explain the Riemann integrability of

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx$$

9. (a) $\int_0^\infty \frac{dx}{1+x^2}$ is an improper integral. Justify why it is an improper integral. 1

(b) Write the statement of Abel's test. 1

(c) Show that

$$\int_0^\infty \sin x^2\,dx$$

is convergent. 4

Or

Test the convergence of the integral

$$\int_0^\infty \frac{\cos x}{1+x^2}\,dx$$

10. (a) Show that $B(m, n) = B(n, m)$. 2

(b) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. 2

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