3 SEM TDC MTH M 2

2016

(November)

MATHEMATICS

(Major)

Course : 302

(Coordinate Geometry and Algebra—I)

Full Marks : 80

Pass Marks : 32 (Backlog)/24 (2014 onwards)

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Coordinate Geometry)

SECTION—I

(2-Dimension)

(*Marks* : 27)

1. *(a)* Is there any change on the degree of the curve after transformation of axes? (Write Yes or No) 1

(b) How would you find the angle through which the axes are rotated keeping the origin fixed for the expression $ax^2 + 2hxy + by^2$? 2

(c) Show that, by orthogonal transformation without change of origin, $g^2 + f^2$ remains unchanged in the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$ 4

Or

Prove that the distance between the points (x_1, y_1) and (x_2, y_2) is unaltered by rotation of axes. 4

2. (a) Prove that the equation $\sqrt{3}x^2 - 2xy - \sqrt{3}y^2 + 4 = 0$ is transformed to $xy = 1$, when the rectangular axes are turned through an angle $\frac{\pi}{6}$ about its origin as centre. 4

(b) If the two pairs of the lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair, then prove that $pq + 1 = 0$.

Or

Show that the straight lines represented by the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

will be equidistant from the origin if

$$f^4 - g^4 = c(bf^2 - ag^2)$$ 3

(c) Show that the equation of the lines through the origin, each of which makes an angle α with the line $y = x$ is $x^2 - 2xy \sec 2\alpha + y^2 = 0$. 3

Or

Show that the area of the triangle formed by the lines of $ax^2 + 2hxy + by^2 = 0$ and $lx + my = 1$ is

$$\frac{\sqrt{h^2 - ab}}{am^2 - 2hlm + bl^2}$$ 3

3. (a) Define conjugate diameters. 1

(b) Prove that the general equation of the second degree represents a conic section. 4

Or

Prove that the equation $m(x^3 - 3xy^2) + y^3 - 3x^2y = 0$ represents three straight lines equally inclined to each other. 4

(c) Reduce the equation

$$14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$$

to the standard form. 5

Or

Find the lengths of the semi-axes of the conic $ax^2 + 2hxy + by^2 = d$. 5

SECTION—II

(3-Dimension)

(*Marks* : 18)

4. (a) Prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$, where α, β, γ are the angles made by a line with the coordinate axes. 1

(b) Find the equation of the plane passing through the point $(2, 5, -8)$ and perpendicular to each of the planes $2x - 3y + 4z + 1 = 0$ and $4x + y - 2z + 6 = 0$. 4

(c) A variable plane makes intercepts on the coordinate axes, the sum of whose squares is k^2 (constant). Show that the locus of the foot of the perpendicular from the origin to the plane is

$$\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right)(x^2 + y^2 + z^2)^2 = k^2$$

5

Or

Prove that

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ and } \frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$$

are coplanar. Also find the equation of the plane. 5

5. (a) Define skew lines. Mention whether skew lines are coplanar or not. 1+1=2

(b) Show that the shortest distance between the lines $x+a=2y=-12z$ and $x=y+2a=6(z-a)$ is $2a$. 3

(c) Find the length and position of the shortest distance between the lines

$$\frac{x}{4}=\frac{y+1}{3}=\frac{z-2}{2}$$

$$5x-2y-3z+6=0$$

$$x-3y+2z-3=0$$ 3

Or

Find the length of the shortest distance between the lines

$$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}, \quad \frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$$

Also find its equation and the points where it intersects the lines. 3

GROUP—B

(Algebra—I)

(*Marks* : 35)

6. (*a*) What do you mean by an algebraic structure? 1

(*b*) Write down the difference between relations and functions with the help of examples. 1

(*c*) Show that the identity element in a group is unique. 2

(*d*) Answer any *two* of the following questions : 3×2=6

(*i*) If $a, b, c \in G$ (a group), then show that

$ab = ac \Rightarrow b = c$ and $ba = ca \Rightarrow b = c$

(*ii*) Show that the set $G = \{1, \omega, \omega^2\}$ is a group with respect to multiplication, where ω is an imaginary cube root of unity.

(*iii*) Prove that every cyclic group is an Abelian group.

7. Answer any *two* of the following questions : 5×2=10

(*a*) Prove that if G is an Abelian group, then for all $a, b \in G$ and all integers n, $(ab)^n = a^n b^n$.

(b) If H_1 and H_2 are two subgroups of a group G, then $H_1 \cap H_2$ is also a subgroup of G. What about $H_1 \cup H_2$? Justify.

(c) Define cyclic group with example. Find the number of generators of the cyclic group G of order 8.

8. (a) Cosets are empty sets. State true or false. Give reasons. 1+1=2

(b) Prove that the order of each subgroup of a finite group is a divisor of the order of the group. 3

9. Answer any *two* of the following questions : 5×2=10

(a) Define normal subgroup. Prove that the intersection of any two normal subgroups of a group is a normal subgroup.

(b) If f is a homomorphism of a group G into a group G' with kernel K, then show that K is a normal subgroup of G.

(c) Prove that every homomorphic image of a group G is isomorphic to some quotient group of G.

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