Total No. of Printed Pages—8

3 SEM TDC MTH M 2

2014

(November)

MATHEMATICS

(Major)

Course : 302

(Coordinate Geometry and Algebra—I)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Coordinate Geometry)

SECTION—I

(2-Dimension)

(*Marks* : 27)

1. *(a)* Find out the angle between the pair of lines represented by the equation

$$6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$$ 1

(b) Choose a new origin (h, k), without changing the directions of the axes such that the equation

$$5x^2 - 2y^2 - 30x + 8y = 0$$

may reduce to the form $Ax'^2 + By'^2 = 1$. 2

(c) Transform the equation

$$x^2 + 2xy \tan 2\alpha - y^2 = a^2 \sec 2\alpha$$

to rectangular axes inclined at angle α to the old rectangular axes. 3

Or

Find the angle through which the axes be rotated so that the expression $ax^2 + 2hxy + by^2$ may become of the form $a'x'^2 + b'y'^2$.

2. (a) Find the value of λ so that the equation

$$2x^2 + xy - y^2 - 11x - 5y + \lambda = 0$$

may represent a pair of lines. 3

(b) Find the equation of the pair of lines through the origin which represents the lines perpendicular to the pair of lines

$$ax^2 + 2hxy + by^2 = 0$$

3

Or

Show that the angle between one of the lines given by $ax^2 + 2hxy + by^2 = 0$ and one of the lines

$$ax^2 + 2hxy + by^2 + \lambda(x^2 + y^2) = 0$$

is equal to the angle between the other two lines of the systems.

(c) Prove that the straight lines represented by the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

will be equidistant from the origin if

$$f^4 - g^4 = c(bf^2 - ag^2)$$ 5

Or

If

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of lines, prove that the area of the triangle formed by these lines and the x-axis is

$$\frac{g^2 - ac}{a\sqrt{h^2 - ab}}$$

3. (a) State True or False : 1

When the focus lies on the directrix, the conic section is a pair of lines.

(b) Find the equation of the chord of contact of tangents from a point (x_1, y_1) to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$ 4

Or

Find the condition that the pair of lines $Ax^2 + 2Hxy + By^2 = 0$ may be conjugate diameters of the conic

$$ax^2 + 2hxy + by^2 = 1$$

(c) Reduce the following equation to the standard form :

$$3x^2 - 6xy - 5y^2 - 6x + 22y - 17 = 0$$ 5

Or

Prove that every Cartesian equation of the second degree, i.e.

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a conic.

SECTION—II

(3-Dimension)

(*Marks* : 18)

4. (*a*) Write the direction cosines of the line joining the origin and the point (1, 2, −3). 1

(*b*) Find the equation of the plane which passes through the intersection of the planes $x-2y-3z=4$, $2x+3y-z=1$ and is perpendicular to the plane $3x-y+2z+5=0$. 4

(*c*) Find the distance of the point (1, −2, 3) from the plane $x-y+z=5$ measured parallel to the line

$$\frac{x}{2}=\frac{y}{3}=\frac{z-1}{-6}$$ 5

Or

Show that the lines

$$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1} \text{ and } \frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$$

intersect. Find the coordinates of the point of intersection.

5. (a) What is the shortest distance between two intersecting straight lines? 1

(b) Find the shortest distance between the line

$$ax + by + cz + d = 0 = a'x + b'y + c'z + d'$$

and the z-axis. 2

(c) Find the length and the equations of the shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and $$\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$ 5

Or

Prove that the shortest distance between the lines

$$\frac{x-3}{1} = \frac{y-4}{1} = \frac{z+1}{-3} \text{ and } \frac{x-1}{-1} = \frac{y-3}{3} = \frac{z-1}{2}$$

is $\frac{15}{\sqrt{138}}$ and the equations of the shortest distance are

$$7x - 37y - 10z + 117 = 0 = 5x + 13y - 17z - 27$$

GROUP—B

(Algebra—I)

(*Marks* : 35)

6. (*a*) If a set A has n members, then state the number of binary compositions on A. 1

(*b*) The set of integers, with respect to usual multiplication does not form a group. Justify it. 2

(*c*) State True or False : 1

A group of prime order is Abelian.

(*d*) Answer any *two* questions : 3×2=6

(*i*) Show that the centre of a group G is a subgroup of G.

(*ii*) If G is a group in which $(ab)^i = a^i b^i$ for three consecutive integers i and any a, b in G, then show that G is Abelian.

(*iii*) Prove that an infinite cyclic group has precisely two generators.

7. Answer any *two* questions : 5×2=10

(*a*) Let G be a group. Suppose $a, b \in G$ such that—

(*i*) $ab = ba$;

(*ii*) $(o(a), o(b)) = 1$.

Show that $o(ab) = o(a)o(b)$.

(b) If H and K are finite subgroups of a group G, then prove that

$$o(HK) = \frac{o(H)\,o(K)}{o(H \cap K)}$$

(c) If a group has finite number of subgroups, then show that it is a finite group.

8. (a) Define kernel of a group homomorphism. 1

(b) Prove that a subgroup H of a group G is normal in G iff $g^{-1}Hg = H$ for all $g \in G$. 4

9. Answer any *two* questions : 5×2=10

(a) If G is a group such that $\frac{G}{z(G)}$ is cyclic, where $z(G)$ is centre of G, then show that G is Abelian.

(b) Prove that every quotient group of a cyclic group is cyclic.

(c) State and prove fundamental theorem of group homomorphism.

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