1 SEM TDC MTH M 1

2019

(November)

MATHEMATICS

(Major)

Course : 101

(Classical Algebra, Trigonometry and Vector Calculus)

Full Marks : 80
Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Classical Algebra)

1. *(a)* Write the domain of a real sequence. 1

(b) Write the limit point of the sequence $\{2, 4, 2, 6, 2, 8, \ldots\}$. 1

(c) Prove that a sequence cannot converge to more than one limit point. 4

(d) Discuss the convergence of the sequence $\{S_n\}$, where

$$S_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$$ 4

Or

Show that every convergent sequence is bounded.

2. *(a)* Write the Cauchy's general principle of convergence of series. 1

(b) Test the convergence of the series

$$\frac{1}{1\cdot2\cdot3} + \frac{3}{2\cdot3\cdot4} + \frac{5}{3\cdot4\cdot5} + \frac{7}{4\cdot5\cdot6} + \ldots$$ 4

(c) State and prove the necessary condition for convergence of an infinite series. 5

Or

Show that the series $\sum \frac{1}{n}$ does not converge.

(d) Show that the series

$$1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots$$

is convergent. 5

3. *(a)* Write the complex roots of the equation $x^3 - 1 = 0$. 1

(b) Write the nature of the roots of the equation $x^3 + px + q = 0$, if p and q are positive. 2

(c) Find the sum of the fourth powers of the roots of the equation $x^3 - x - 1 = 0$. 3

(d) If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, then find the equation whose roots are $\alpha - \frac{1}{\beta\gamma}$, $\beta - \frac{1}{\alpha\gamma}$, $\gamma - \frac{1}{\alpha\beta}$. 4

Or

Find the equation whose roots are the square of the roots of the equation $2x^3 - 3x^2 + 4x - 5 = 0$.

(e) Solve $x^3 - 3x + 1 = 0$ by using Cardan's method. 5

Or

If α, β, γ are the roots of the equation $x^3 + ax^2 + bx + c = 0$, then find the value of $\sum \alpha^2 \beta$ in terms of the coefficients of the equation.

GROUP—B

(Trigonometry)

4. *(a)* Write the value of

$$\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$$ 1

(b) Express $\left(\frac{\cos x + i\sin x}{\sin x + i\cos x}\right)^6$ in the form $x + iy$. 3

(c) Solve the equation $x^7 + x^4 + x^3 + 1 = 0$ by using De Moivre's theorem. 4

Or

If $x = \cos\alpha + i\sin\alpha$, $y = \cos\beta + i\sin\beta$, $z = \cos\gamma + i\sin\gamma$ and $x + y + z = 0$, then show that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

5. *(a)* Show that

$$\log i = \left(2n + \frac{1}{2}\right)\pi i$$ 2

(b) If $\tan\log(x+iy)=a+ib$, $a^2+b^2\neq 1$, then prove that

$$\tan\log(x^2+y^2)=\frac{2a}{1-a^2-b^2}$$ 3

Or

Show that $\sin(\log i^i)=-1$.

6. (a) Show that

$$\log\sec\theta=\frac{1}{2}\tan^2\theta-\frac{1}{4}\tan^4\theta+\frac{1}{6}\tan^6\theta-\ldots$$

if $|\theta|<\frac{\pi}{4}$. 4

Or

(b) Show that

$$\left(\frac{2}{3}+\frac{1}{7}\right)-\frac{1}{3}\left(\frac{2}{3^3}+\frac{1}{7^3}\right)+\frac{1}{5}\left(\frac{2}{3^5}+\frac{1}{7^5}\right)-\cdots=\frac{\pi}{4}$$

7. (a) Write the sum of the series $\cos x+\cos 2x+\cos 3x+\cdots+\cos nx$. 1

(b) Find the sum of the series $\sin\alpha-\sin 2\alpha+\sin 3\alpha-\cdots$ to n terms. 3

(c) Separate into real and imaginary parts of (any *one*) : 4

(i) $\cosh(x+iy)$

(ii) $\sin^{-1}(x+iy)$

GROUP—C

(Vector Calculus)

8. (a) $$\frac{d}{dt}(\vec{A}\times\vec{B})=\vec{A}\times\frac{d\vec{B}}{dt}+\vec{B}\frac{d\vec{A}}{dt}$$

Write the above statement correctly. 1

(b) Find the value of $\nabla\cdot\vec{r}$, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$. 1

(c) Determine the unit tangent vector at any point on the curve $\vec{r}=t^2\hat{i}+t^2\hat{j}-\frac{t^2}{2}\hat{k}$. 2

(d) Let $\vec{A}=x^2y^2\hat{i}+y^2z^2\hat{j}+x^2z^2\hat{k}$. Find $\nabla\times\vec{A}$ at $(-1, 1, 1)$. 3

(e) Find the directional derivative of $\phi=x^2y^2z+2xy^2z$ at $P(1, 2, 1)$ in the direction of $2\hat{i}+\hat{j}+2\hat{k}$. 4

Or

Show that curl grad $\phi=0$

(f) Show that

$$\nabla \cdot (\phi \vec{A}) = (\nabla\phi \cdot \vec{A}) + \phi(\nabla \cdot \vec{A})$$

where ϕ is a scalar function and $\vec{A}$ is a vector function. 4

Or

Show that

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

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