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**1 SEM TDC MTMH (CBCS) C1**

**2019**

( December )

**MATHEMATICS**

( Core )

Paper : C-1

( **Calculus** )

Full Marks : 60

Pass Marks : 24

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

1. (a) Write the value of  $\frac{d}{dx}(\cosh x)$ . 1
- (b) Let  $f(x)$  is a differentiable function on an open interval  $I$ , Write when it is concave up. 1
- (c) Find  $\frac{d}{dx}(\tanh\sqrt{1+x^2})$ . 2

- (d) Find  $y_n$  (any one) : 3  
 (i)  $y = \sin^2 x \cos^2 x$   
 (ii)  $y = \sin x \sin 2x \sin 3x$
- (e) If  $y = x^2 \tan^{-1} x$ , then find  $y_n$ . 4

Or

If  $y = (\sin^{-1} x)^2$ , then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$$

- (f) If  $y = e^{ax+b} \sin x$ , then find  $y_n$ . 3
- (g) Evaluate (any one) : 3  
 (i)  $\lim_{x \rightarrow 0} \frac{\tan x - x}{x - \sin x}$   
 (ii)  $\lim_{x \rightarrow \infty} \frac{x^2 + 3x}{1 - 5x^2}$

- (h) Find the minimum value of the function

$$f(x) = 1 + 2 \sin x + 3 \cos^2 x, \quad x \in \left[0, \frac{\pi}{2}\right] \quad 3$$

2. (a) Show that

$$\int_0^{\frac{\pi}{2}} \sin^n x \, dx = \int_0^{\frac{\pi}{2}} \cos^n x \, dx \quad 1$$

- (b) Obtain the reduction formula for

$$\int \sin^n x \, dx, \quad n > 1$$

and hence write the value of  $\int \sin^3 x \, dx$ .

$$4+1=5$$

(c) Evaluate (any one) : 4

(i)  $\int \tan^5 x \, dx$

(ii)  $\int \sec^5 x \, dx$

(d) A region bounded by the curve  $y = x^2 + 1$  and the line  $x + y = 3$  is revolved about the  $x$ -axis to generate a solid. Find the volume of the solid. 5

Or

A region is enclosed by the triangle with vertices  $(1, 0)$ ,  $(2, 1)$ ,  $(1, 1)$ . Find the volume of the solid generated by revolving the region about the  $y$ -axis.

3. (a) Write the equation of parabola in polar form. 1

(b) Find an equation for the hyperbola with eccentricity  $\frac{5}{3}$  and directrix  $x = 3$  in polar form. 2

(c) Find the parametric equations and a parameter interval for the motion of a particle that starts at  $(a, 0)$  and traces the circle  $x^2 + y^2 = a^2$ , once clockwise. 2

(d) Determine the nature of the conic represented by

$$x^2 + 2xy + y^2 + 2x - y + 2 = 0 \quad 2$$

- (e) Determine the angle of rotation of axes in order to remove the  $xy$  term from

$$3x^2 - 2\sqrt{3}xy + y^2 = 1 \quad 2$$

- (f) Find a parameterization for the line segment with end points  $(-1, -3)$  and  $(4, 1)$ . 3

Or

Find perimeter of a circle of radius  $a$  defined parametrically by  $x = acost$ ,  $y = asint$ ,  $0 \leq t \leq 2\pi$ .

- (g) Find the area enclosed by the ellipse  $x = acost$ ,  $y = bsint$ ,  $0 \leq t \leq 2\pi$ . 3

Or

Find the area of the surface generated by revolving the curves  $x = cost$ ,  $y = 2 + sint$ ,  $0 \leq t \leq 2\pi$ , about  $x$ -axis.

4. (a) Let  $\vec{r}(t) = (\cos t)\hat{i} + (\tan t)\hat{j} + t\hat{k}$ . Find

$$\lim_{t \rightarrow \frac{\pi}{4}} \vec{r}(t) \quad 2$$

- (b) If  $\vec{r}(t) = (4 \cos t)\hat{i} + (4 \sin t)\hat{j} + (\cos^2 t)\hat{k}$  be the position vector of a particle at any time  $t$ , then find velocity at any time  $t$ . 2

(c) Let  $\vec{r}(t)$  is a differentiable vector function of  $t$  of constant length. Show that  $\vec{r}(t)$  and  $\vec{r}'(t)$  are orthogonal. 2

(d) Evaluate

$$\int_0^{\pi} [(\cos t)\hat{i} + 2t\hat{j} + 3t^2\hat{k}] dt \quad 3$$

Or

Find the scalar triple product of the vectors  $\hat{i} + 2\hat{j} + 3\hat{k}$ ,  $\hat{i} + 3\hat{j} + 5\hat{k}$  and  $\hat{i} + \hat{j} + 6\hat{k}$ .

(e) Write the value of  $\vec{a} \times (\vec{b} \times \vec{c})$ . 1

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