**1 SEM TDC MTH M 1**

**2 0 1 8**

( November )

# MATHEMATICS

( Major )

Course : 101

**( Classical Algebra, Trigonometry and Vector Calculus )**

*Full Marks* : 80

*Pass Marks* : 32/24

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

GROUP—A

**( Classical Algebra )**

**1.** *(a)* Write the limit points of the sequence

$$S_n = (-1)^n \left(1 + \frac{1}{n}\right)$$ 1

*(b)* Define a Cauchy sequence. Prove that the sequence $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$ is a Cauchy sequence. 1+2=3

*(c)* Show that the sequence $\{S_n\}$ given by

$$S_n = \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{n!}, \quad \forall\, n \in \mathbb{N}$$

is convergent. 3

*(d)* Show that every bounded sequence has a limit point. 3

**2.** *(a)* Show that the series $1 + 2 + 3 + \ldots + n + \ldots$ cannot converge. 2

*(b)* Prove that if $\Sigma u_n$ is a series of positive terms and $\Sigma u_n$ is convergent, then

$$\Sigma \frac{u_n}{1 + u_n}$$

is convergent. 3

*(c)* Write the statement of Cauchy's Root Test for the convergence of a series of positive terms and test the convergence of the series

$$1+\frac{x}{2}+\frac{x^2}{3^2}+\frac{x^3}{4^3}+\ldots\infty, \; x>0$$

applying it. 2+3=5

*(d)* Test the convergence of any *one* of the following : 5

*(i)* $\sum_{n=1}^{\infty} \frac{1}{2^n+3^n}$

*(ii)* $\sum \frac{n^3}{n^3+1} x^{n-1}$

3. *(a)* If the roots of the equation

$$x^4+px^3+qx^2+rx+s=0$$

are connected by the relation $\beta+\gamma=\alpha+\delta$, where $\alpha$, $\beta$, $\gamma$, $\delta$ are the roots of the equation, then prove that

$$p^3-4pq+8r=0$$ 4

*(b)* If the roots of the equation $x^n - 1 = 0$ be $1, r_1, r_2, \ldots, r_{n-1}$, then prove that

$$(1-r_1)(1-r_2)\ldots(1-r_{n-1}) = n$$ 3

*(c)* Find the equation whose roots are the roots of the equation $x^5 + 4x^3 - x^2 + 11 = 0$, each diminished by 3. 3

*(d)* Solve by Cardan's method : 5

$$x^3 - 21x - 344 = 0$$

GROUP—B

( **Trigonometry** )

**4.** Answer any *two* : 4×2=8

*(a)* Find the equation whose roots are the $n$th powers of the roots of the equation

$$x^2 - 2x\cos\theta + 1 = 0$$

*(b)* Prove that

$$\sin 5x = 16\sin^5 x - 20\sin^3 x + 5\sin x$$

*(c)* Deduce the expansion of $\cos\alpha$ in ascending powers of $\alpha$.

**5.** *(a)* Show that

$$\log i = \left(2n + \frac{1}{2}\right) i\pi$$ 2

*(b)* If $\tan\log(x+iy) = a+ib$, where $a^2 + b^2 \neq 1$, prove that

$$\tan\log(x^2+y^2) = \frac{2a}{1-a^2-b^2}$$ 3

**6.** *(a)* Write the interval of $\theta$ for which

$$\theta = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta - \dots$$ 1

*(b)* Show that

$$\pi = 2\sqrt{3}\left[1 - \frac{1}{3.3} + \frac{1}{5.3^2} - \frac{1}{7.3^3} + \dots\right]$$ 3

**7.** *(a)* Write the condition that is to be satisfied by the common difference of the angles in AP so that the sum of the sines and sum of the cosines of $n$ angles are each equal to zero. 2

*( Turn Over )*

(b) Answer any *two* : 3×2=6

(i) Sum to $n$ terms of the series

$$\cos^2\alpha + \cos^2 3\alpha + \cos^2 5\alpha + \ldots$$

(ii) If $\sin(\alpha + i\beta) = x + iy$, prove that

$$x^2\operatorname{cosec}^2\alpha - y^2\sec^2\alpha = 1$$

(iii) Separate $\sinh(x+iy)$ into real and imaginary parts.

GROUP—C

**( Vector Calculus )**

8\. (a) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where $t$ is the time. Find the component of velocity at time $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. 4

(b) Find a unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$. 4

(c) Define directional derivative of a scalar field $\phi$ in the direction $\vec{a}$. 2

(d) Prove that the vector

$$\vec{A} = 3y^4z^2\hat{i} + 4x^3z^2\hat{j} - 3x^2y^2\hat{k}$$

is solenoidal. 2

(e) Find curl $\vec{A}$ at the point (1, −1, 1) if

$$\vec{A} = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$$ 3

★★★