1 SEM TDC MTH M 1

2017

(November)

MATHEMATICS

(Major)

Course : 101

(Classical Algebra, Trigonometry and Vector Calculus)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Classical Algebra)

1. *(a)* Write the range of the sequence

$$\{S_n\} = \{(-1)^n,\ n \in N\}$$ 1

(b) Write the limit point of the sequence

$$\{S_n\} = \left\{\frac{1}{n},\ n \in N\right\}$$ 1

(c) Write the necessary and sufficient condition for the convergence of a monotonic sequence. 1

(d) Write two properties of a convergent sequence. 2

(e) Show that

$$\lim \frac{1+2+3+\ldots+n}{n^2} = \frac{1}{2}$$ 2

(f) Prove that every convergent sequence is bounded. 3

Or

Show that the sequence

$$\{S_n\} = \left\{1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}\right\}$$

cannot converge.

2. (a) Write Cauchy's general principle of convergence for series. 1

(b) If the series $\sum_{n=1}^{\infty} u_n$ is divergent, then write the nature of the series $\sum_{n=0}^{\infty} u_n$. 1

(c) Define an alternating series. 1

(d) Write the name of a test for testing the convergence of a series when d'Alembert's ratio test for convergence of the series fails. 1

(e) Write the statement of d'Alembert's ratio test. 2

(f) Test the convergence of the series

$$\sum \frac{n^2-1}{n^2+1} x^n, \quad x > 0$$ 4

(g) Prove that a positive term series $\sum \frac{1}{n^p}$ is convergent if and only if $p > 1$. 5

Or

Test the convergence of the sequence

$$1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots$$

3. (a) Let

$$P_0 x^n + P_1 x^{n-1} + P_2 x^{n-2} + \ldots + P_n = 0,$$
$$P_i, \; i = 0, 1, 2, \ldots, n$$

are real constants. Write the value of the sum of the roots of the equation. 1

(b) Write the equation whose roots are the reciprocals of the roots of the equation

$$2x^5 - x^3 + 11x - 6 = 0$$ 2

(c) Solve the equation

$$x^3 - 14x^2 - 84x + 216 = 0,$$

given that the roots are in geometrical progression. 3

(d) If α, β, γ be the roots of the equation $x^3+px+q=0$, form the equation whose roots are $\beta+\gamma-\alpha$, $\gamma+\alpha-\beta$, $\alpha+\beta-\gamma$. 4

Or

If α, β, γ be the roots of the equation $x^3+px+q=0$, find the value of $\sum\frac{1}{(\beta+\gamma)^2}$ in terms of the coefficients.

(e) Solve $x^3+15x-124=0$ by using Cardan's method. 5

Or

Find the equation whose roots are the squares of the roots of the equation

$$2x^3-3x^2+4x-5=0$$

GROUP—B

(Trigonometry)

4. *(a)* $(\cos m\theta+i\sin m\theta)^n=(\cos n\theta+i\sin n\theta)^m$. State true or false. 1

(b) Find all the values of $(1+i)^{\frac{1}{5}}$. 3

Or

If $x = \cos\alpha + i\sin\alpha$, $y = \cos\beta + i\sin\beta$, $z = \cos\gamma + i\sin\gamma$ and if $x + y + z = 0$, then show that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

(c) Express $\sin 5x$ in powers of $\sin x$. 4

Or

Prove that

$$(a + ib)^{\frac{m}{n}} + (a - ib)^{\frac{m}{n}} = 2(a^2 + b^2)^{\frac{m}{2n}} \cos\left(\frac{m}{n}\tan^{-1}\frac{b}{a}\right)$$

5. (a) Write for what values of x, the expression $e^{ix} = \cos x + i\sin x$ is true. 1

(b) Show that $\sin(\log i^i) = -1$. 4

Or

Express $(x + iy)^i$ in the form $A + iB$.

6. (a) Write the interval of θ for which the Gregory's series

$$\theta = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta - \ldots$$

is valid. 1

(b) Show that

$$\frac{\pi}{2} = \sqrt{3}\left(1 - \frac{1}{3 \cdot 3} + \frac{1}{5 \cdot 3^2} - \frac{1}{7 \cdot 3^3} + \dots\right)$$ 3

Or

Show that

$$\tan^{-1}\frac{1-\cos\theta}{1+\cos\theta} = \tan^2\frac{\theta}{2} - \frac{1}{3}\tan^6\frac{\theta}{2} + \frac{1}{5}\tan^{10}\frac{\theta}{2} - \dots,$$

where $0 < \theta < \frac{\pi}{2}$.

7. *(a)* Write the sum of the series

$$\cos\theta + \cos 2\theta + \cos 3\theta + \dots + \cos n\theta$$ 1

(b) Find the sum of the series

$$\sin^2\theta + \sin^2 2\theta + \sin^2 3\theta + \dots + \sin^2 n\theta$$ 3

(c) Find the sum of the series to n terms,

$$\cos\theta\cos 2\theta + \cos 3\theta\cos 4\theta + \cos 5\theta\cos 6\theta + \dots$$ 4

Or

Separate $\cos^{-1}(x+iy)$ into real and imaginary parts.

GROUP—C

(Vector Calculus)

8. (a) Define vector differential operator del. 1

(b) Define solenoidal vector. 1

(c) Let $\vec{A} = xy\hat{i} + yz\hat{j} + e^{xyz}\hat{k}$. Show that

$$\frac{\partial \vec{A}}{\partial x} \neq \frac{\partial \vec{A}}{\partial y}$$ 2

(d) Let $\vec{a}$ has constant magnitude and $\left|\frac{d\vec{a}}{dt}\right| \neq 0$. Show that $\vec{a}$ and $\frac{d\vec{a}}{dt}$ are perpendicular. 3

(e) Show that

$$\nabla \cdot (\phi\vec{A}) = (\nabla\phi) \cdot \vec{A} + \phi(\nabla \cdot \vec{A})$$ 4

Or

Evaluate $\nabla \cdot (r^3\vec{r})$.

(f) Find $\nabla\left(\frac{1}{r}\right)$, $r = |\vec{r}|$, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. 4

Or

Prove that $\text{div curl } \vec{A} = 0$.

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