1 SEM TDC MTH M 1

2013

(November)

MATHEMATICS

(Major)

Course : 101

(Classical Algebra, Trigonometry and Vector Calculus)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Classical Algebra)

1. *(a)* Choose the correct answer for the following : 1

A sequence is bounded if and only if

(i) its domain is bounded

(ii) its range is bounded

(iii) it is bounded above

(iv) it is bounded below

(b) Write the bounds of the sequence $\{S_n\}$, where $S_n = 1 + (-1)^n$, $n \in \mathbb{N}$. 2

(c) Show that the sequence $\{S_n\}$, where

$$S_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}$$

is a bounded sequence. 2

(d) Show that a sequence converges to one limit point only. 5

Or

Show that every bounded sequence with a unique limit point is convergent.

2. (a) Write when an infinite series diverges. 1

(b) Write the statement of Leibnitz test for convergence of an alternating series. 2

(c) Test the convergence of the infinite series $\Sigma \frac{1}{n}$. 4

(d) Test the convergence of the infinite series $1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \cdots$. 4

(e) Show that if a series ΣU_n of positive monotonic decreasing terms converges, then $nU_n \to 0$ as $n \to \infty$. 4

Or

Test the convergence of the infinite series $1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + \cdots$.

3. (a) If α be a root of an equation $f(x)=0$, then write a factor of $f(x)$. 1

(b) If $-\sqrt{3}$ and $-(1+i\sqrt{6})$ are two roots of the equation $x^4+2x^3+4x^2-6x-21=0$, then write the other two roots of the equation. 1

(c) Write the nature of the roots of the equation $x^3-5x^2-4x+20=0$. 2

(d) If $-1, -1, -4$ are the roots of the equation $x^3+6x^2+9x+4=0$, then write the equation whose roots are $-2, -2, -8$. 2

(e) If α, β, γ be the roots of the equation $x^3+px^2+qx+r=0$, then find the value of

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}$$ 3

(f) Solve the equation $x^3-21x-344=0$ by Cardan's method. 6

Or

If α, β, γ be the roots of the equation $x^3+px^2+qx+r=0$, then find the value of $\alpha^4+\beta^4+\gamma^4$.

GROUP—B

(Trigonometry)

4. (a) Choose the correct answer : 1

$(\cos x + i\sin x)^{\frac{3}{4}}$ is equal to

(i) $\cos\frac{4}{3}x + i\sin\frac{3}{4}x$

(ii) $\cos\frac{3}{4}x - i\sin\frac{3}{4}x$

(iii) $\cos\frac{3}{4}x + i\sin\frac{4}{3}x$

(iv) $\cos\frac{3}{4}x + i\sin\frac{3}{4}x$

(b) Write the roots of the quadratic equation $x^2 - 2x\cos\theta + 1 = 0$. 2

(c) Express $\cos 5\theta$ in powers of $\cos\theta$. 5

Or

If $x = \cos\theta + i\sin\theta$ and $1 + \sqrt{1 - a^2} = na$, prove that $1 + a\cos\theta = \frac{a}{2n}(1 + nx)(1 + \frac{n}{x})$.

5\. (a) Show that $\log i = (2n + \frac{1}{2})\pi i$. 2

(b) Show that

$$\log(1 + \cos 2\theta + i \sin 2\theta) = \log(2\cos\theta) + i\theta.$$ 3

Or

Express $\cos(x + iy)$ in the form $A + iB$.

6\. (a) Write the interval of x for which $\tan^{-1} x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \cdots +$

$$(-1)^{n-1}\frac{x^{2n-1}}{2n-1} + \cdots$$ 1

(b) Show that

$$\frac{\pi}{4} = \left(\frac{2}{3} + \frac{1}{7}\right) - \frac{1}{3}\left(\frac{2}{3^3} + \frac{1}{7^3}\right) + \frac{1}{5}\left(\frac{2}{3^5} + \frac{1}{7^5}\right) - \cdots$$ 3

Or

Expand $\log \cos x$ in ascending powers of $\tan x$, $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$.

7\. (a) Choose the correct answer : 1

$\sin(ix)$ is equal to

(i) $i \sin hx$

(ii) $\sin hx$

(iii) $-i \sin hx$

(iv) $i \cos hx$

(b) Write the period of $\tan hx$. 1

(c) Write the expansion of $\cos hx$ in terms of x. 2

(d) Find the sum of

$\cos\theta\cos 2\theta + \cos 3\theta\cos 4\theta + \cos 5\theta\cos 6\theta + \cdots$

up to n terms. 4

Or

Separate $\tan h(x+iy)$ into real and imaginary parts.

GROUP—C

(Vector Calculus)

8. (a) Choose the correct answer : 1

A vector $\vec{V}$ is a solenoidal vector of

(i) $\nabla \times \vec{V} = 0$

(ii) $\nabla \cdot \vec{V} = 0$

(iii) $\nabla |\vec{V}| = 0$

(iv) $\nabla \cdot \vec{V} =$ non-zero constant

(b) Find curl $\vec{A}$, where $\vec{A} = x\hat{i} + z\hat{j} + 2y\hat{k}$. 2

(c) Show that $\nabla \times (\phi \vec{A}) = (\nabla \phi) \times \vec{A} + \phi(\nabla \times \vec{A})$, where ϕ is a scalar function and $\vec{A}$ is a vector function. 5

Or

Show that $\nabla f(r) = \frac{f'(r)}{r} \vec{r}$, where $r = |\vec{r}|$, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(d) Show that $\nabla \phi$ is a vector perpendicular to the surface $\phi(x, y, z) = c$, where c is a constant. 3

(e) Evaluate $\nabla \cdot (r^3 \vec{r})$, $r = |\vec{r}|, \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. 4

Or

Find the directional derivative of $\phi = 4xz^2 - 3xyz$ at $(1, -1, 1)$ in the direction of $2\hat{i} - 3\hat{j} + 6\hat{k}$.

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